POWER FACTOR PROBLEMS IN ELECTRICITY SUPPLY

AN ELEMENTARY TREATISE FOR STUDENTS, SUPPLY ENGINEERS AND POWER USERS

BY

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PREFACE

THIS book is intended to give a simple account of power factor, its meaning and the causes of its variation, its effects on the cost of and the charges for electricity supply, and the ways in which it can be artificially changed. Power and lagging magnetising volt-amperes are considered to be objective quantities in an A.C. electric supply, for both of which the consumer is required to pay under what are called power factor tariffs, and the process of power factor correction is explained as one whereby the consumer generates part or all of his magnetising volt-ampere supply instead of purchasing it from the supply authority.

The quantitative aspect of power factor is illustrated by fully-worked numerical examples in Chapters I, II and VI. The important question of the economical limits of power factor correction is dealt with fully in the latter chapter. Electrical measurements connected with power factor are described in a non-technical way in Chapter II.

The mathematics in the main text are restricted to the simplest algebraic symbolism. In the Appendix will be found a discussion of a few theoretical questions concerning power factor that

require some slightly more advanced mathematics for their adequate treatment.

Whilst the book has been written primarily for practical engineers engaged in the production and utilisation of electrical energy, the author hopes that it may be of some assistance to students who are trying to master the difficulties of elementary A.C. theory.

G. W. S.

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POWER FACTOR PROBLEMS IN ELECTRICITY SUPPLY

CHAPTER I

THE MEANING OF POWER FACTOR

Electric Power.

The rate at which electrical energy is consumed in a circuit is called the power. If this rate is measured in units an hour, this gives the power in thousands of watts or kilowatts (kW). A watt of electric power is exactly equivalent to a definite amount of power of a different character; thus 746 watts are equivalent to a mechanical horse-power or a rate of doing mechanical work that is equal to 33,000 ft.-lbs. a minute. The power used in a circuit is thus the most important characteristic of the electric supply to this circuit.

When a circuit carries direct current, the power consumed in it can be measured in two ways. The first way is illustrated in fig. 1. The current in the circuit is measured by an ammeter A, and the pressure applied to the ends of the circuit is measured by a voltmeter V. The product of simultaneous readings of A in amperes and V in

volts gives the value of the power in watts. Alternatively the power may be measured by a single instrument called a wattmeter W, as shown in fig. 2. This instrument has two electrical cir-

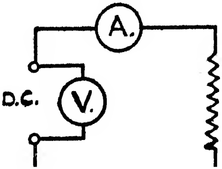

Fig. 1.

cuits which are connected, the one like A and the other like V in fig. 1. The reading of W depends upon the product of amperes by volts and thus gives the power in watts.

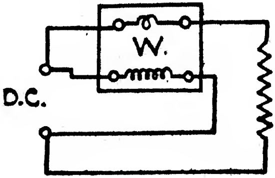

Fig. 2.

The values of the power in a D.C. circuit obtained by these two methods of measurement is the same. If the values determined by instrument readings differ by ever so small an amount then one at least of the instruments is known to be reading incorrectly.

A.C. Power.

The conception of power in a circuit carrying alternating current (A.C.) is much more complex than when the supply is D.C. The values of current and pressure in an A.C. circuit are continually changing, and although the instantaneous value of A.C. power in watts is equal to the product of the corresponding instantaneous values of amperes and volts, it is clear that this instantaneous power value in watts must also be continuously changing. As a matter of fact the instantaneous value of A.C. power oscillates at double the frequency of the current and pressure. The nominal value of A.C. power is the average value taken over a whole cycle of change. This is the value given by a wattmeter connected as shown in fig. 2. Owing to the inertia of the moving system of the instrument the pointer will be unable to follow the rapid changes of instantaneous watts, and it will indicate the average value.

Alternating Currents.

The conception of assigning values of alternating currents or pressures is even more complex, because an A.C. flows similarly in opposite directions during one cycle of change. The average value of any A.C. over such a cycle must therefore be zero. The nominal value of an A.C. in amperes is the

average power loss this current will set up when flowing in a resistance of 1 ohm. Thus 1 ampere of either D.C. or A.C. flowing in 1 ohm sets up 1 watt of power loss. An alternating volt applied to a resistance of 1 ohm will produce 1 ampere of A.C. Alternating values defined in this way are called R.M.S. (root-mean-square) values because the instantaneous power loss in a resistance depends upon the square of the instantaneous value of the current flowing in it.

Wave-Form.

The normal variation of the instantaneous value of an A.C. during one cycle of change is represented

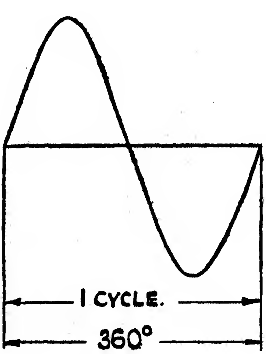

FIG. 3.

graphically by the curve of fig. 3. This is the curve that would represent the movement of the prong

of a vibrating tuning fork, so that an A.C. changing in this way is said to follow the harmonic law. Fig. 3 also represents the manner in which the trigonometrical sine of an angle changes, so that an A.C. following the harmonic law is often said to be sinusoidal. The curve representing the manner of change is called the wave-form of the current. If this is not sinusoidal, it is said to be distorted. The wave-form of the pressure of an A.C. supply generally approximates very closely to the sinusoidal form.

It is shown in books on A.C. theory that when an A.C. is sinusoidal the maximum instantaneous value is $\sqrt{2}$ times the R.M.S. value. This means that an A.C. having maximum positive and negative values in one cycle of $\sqrt{2}$ amperes produces an average power loss of 1 watt in a resistance of 1 ohm.

Average Values. Form Factor.

Although an A.C. has no average value over a whole cycle of change, an average value can be assigned over a half cycle between two zero values. It is shown in the text-books that, for a sinusoidal wave form, this average value is $2/\pi$ times the maximum value (π is the perimeter of a circle of unit diameter).

The ratio R.M.S. value to average value is a characteristic of the wave form, and it is called

the form factor. The form factor of a sinusoidal wave is evidently $1/\sqrt{2}$ divided by $2/\pi$ or $\pi/2\sqrt{2}=1{\cdot}11$ approximately.

Power Factor.

If simultaneous values of R.M.S. amperes and volts and average watts are determined by measurement with an ammeter, a voltmeter and a wattmeter, the product of amperes by volts will bear no certain relation to the watts as it always does in a D.C. circuit. The product of alternating amperes by alternating volts is a quantity different in physical nature from watts and it is called the voltamperes (VA). 1000 VA is called a kilo-voltampere (kVA). The watts in an A.C. circuit may be equal to the VA, cannot be greater, and are generally less. Thus, as we shall study in greater detail hereafter, a current of 10 amperes derived from a 230 volt A.C. supply will involve a power consumption of almost exactly 2300 watts if this power is used in an electric fire. If the supply is used to drive a motor the power may vary from 2300 watts to a value as low as 500. It is possible to have a circuit in which 10 A.C. amperes at 230 volts are flowing, but in which the power used is so small as to be hardly detectable by a wattmeter suitable for measuring the power in an electric fire.

The ratio of watts to VA in an A.C. circuit is called the power factor. It is a measure of the power carried by each VA of the supply. Thus if the power consumption with 10 amperes at 230 volts is 1840 watts, the power factor is 1840/2300 or 0·8. The maximum value of the power factor is unity. The power factor of a special kind of commercial A.C. circuit may be under 0·01.

Power factor is evidently an important characteristic of an A.C. supply. Volts and amperes alone do not determine the power. Knowing the VA and the power factor, the power is known because watts = VA × power factor. Otherwise knowing watts and power factor we can determine the VA because VA = watts/power factor.

We must now study in some detail the physical reasons why the VA in an A.C. circuit, unlike those in a D.C. circuit, can be and often are, ~~less~~ more than the watts.

Reactive Circuits.

When a current flows in a circuit a flux of magnetic lines of force is normally set up, and the amount of the flux depends upon the value of the current. If the circuit consists of a straight wire the flux is relatively small; if the wire is turned back on itself so that the portions carrying current in opposite directions are adjacent, the magnetic effects cancel and the flux is negligible; if the

circuit consists of a helical coil of many turns called a solenoid the flux is greatly increased, and if the solenoïd embraces an iron core the increase of flux is still greater. The total flux per ampere of current is a characteristic property of the circuit.

According to Faraday's law, when the magnetic flux linking with an electric circuit changes, a voltage or electromotive force (e.m.f.) is induced in the circuit, and this e.m.f. is proportional to the rate of change of the flux linkages. This is the principle underlying all methods of producing electrical energy by mechanically driven generators. If a *current carrying circuit* produces magnetic flux, and the current changes, the flux will change also, and this change of flux will set up in the circuit an e.m.f. or voltage additional to that which causes the flow of current. This additional voltage is proportional to the rate at which the flux changes and it is called an e.m.f. of self-induction. The magnitude of the e.m.f. depends upon the rate of change of the current and on a property of the circuit called inductance. Inductance is measured in henrys, and the self-induced voltage is equal to the inductance in henrys multiplied by the rate at which the current changes in amperes per second. Thus, if the inductance of a circuit is $\frac{1}{2}$ henry and the current changes in it at the rate of 20 amperes a second, the self-induced e.m.f. will be $\frac{1}{2} \times 20 = 10$ volts.

This self-induced voltage evidently must oppose the change in the current. If the current is increasing, the self-induced e.m.f. will tend to prevent the increase, otherwise the increase would persist indefinitely, which is inconceivable.

In D.C. circuits, self-induced voltages only occur when changes of the current occur. An A.C. is however continually changing, and if this current flows in a circuit having inductance, a continually changing e.m.f. of self-induction must be present in the circuit opposing the continuous change of the current. Thus, to maintain the continuously changing current, this e.m.f. of self-induction must be opposed.

We have seen that the inductance of a circuit is one of its characteristic properties, and it is possible to have considerable inductance with small resistance. Let us conceive an inductive circuit the resistance of which is negligibly small. If this circuit were connected to a D.C. supply the resulting current would be exceedingly large. It is fairly clear however that if the supply to the circuit is A.C. the actual current will be limited by the e.m.f. of self-induction which, as we have seen, prevents current changes.

We can study this matter in greater detail by considering fig. 4. The dotted curve shows the cycle of current change already given in fig. 3. At the instant represented by A the current is

increasing at its maximum rate. The pressure required to overcome the self-induced e.m.f. must therefore be a maximum and in a direction corresponding to that in which this increase is taking place. At B the current is momentarily at a stationary maximum value so that the induced e.m.f. and hence the requisite supply pressure is

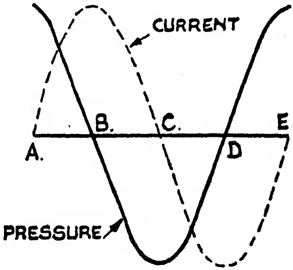

FIG. 4.

zero. At C the current is changing at a maximum rate, but in a direction opposite to that at B. Hence the voltage to overcome the self-induced e.m.f. will be a maximum and opposite to that at A. It follows therefore that the supply pressure required to force an A.C. through an inductive circuit of negligible resistance will vary in a manner represented by the full line in fig. 4, and it is shown in books on A.C. theory that if the dotted curve is sinusoidal, the full-line curve is sinusoidal also. It is seen that the maximum values of the pressure

curve occur $\frac{1}{4}$ cycle before those of the current curve. As the current is the result of the pressure, the variations of current are usually said to occur $\frac{1}{4}$ cycle later or to lag those of the voltage by $\frac{1}{4}$ cycle. This time interval between corresponding maximum values of pressure and current is called a phase difference, and it is often expressed as an angle on the basis of a whole cycle being equal to 360°. Thus the current in an inductive circuit without resistance is said to lag the alternating voltage producing it by 90°.

Now the current in such a circuit can produce no power. The circuit has no resistance and consequently no heat can be produced. Further there is no production of mechanical or any other form of power. The current is said to be wattless, and as the power is zero, the power factor is zero also.

Reactance.

We can, on the assumption that the pressure required to overcome the e.m.f. of self-induction is, like the current, sinusoidal, readily calculate the magnitude of the pressure per R.M.S. ampere of current, by using a little algebraic symbolism. Suppose that the R.M.S. value of the current wave in fig. 3 is 1 ampere; the maximum value in each direction is $\sqrt{2}$ amperes. Consider the portion of the wave between B and D. The total change of

the current is from $\sqrt{2}$ forward to $\sqrt{2}$ reverse or $2\sqrt{2}$ amperes. This change takes place in $\frac{1}{2}$ cycle. If f is the frequency in cycles per second, there will be $2f$ half-cycles a second, and the time for a $\frac{1}{2}$ cycle is $1/2f$ seconds. Thus the *average* rate of change of current in the time interval BD is $2\sqrt{2}$ divided by $1/2f$ or $4\sqrt{2}f$ amperes a second, and if L is the inductance of the circuit in henrys, the *average* value of the self-induced e.m.f. and hence of the supply pressure is $4\sqrt{2}f\mathrm{L}$ volts. Assuming that the self-induced e.m.f. is sinusoidal, the R.M.S. value of the supply pressure per ampere of current will be $4\sqrt{2}f\mathrm{L}$ multiplied by the form factor $\pi/2\sqrt{2}$, or $2\pi f\mathrm{L}$ volts.

This quantity $2\pi f\mathrm{L}$, the R.M.S. voltage for each R.M.S. ampere of current is called the reactance, and it is denoted by the symbol X. The voltage V in a resistance-free inductive circuit is therefore IX, I standing for current. This relation is like Ohm's law, V=IR where R is resistance. X however, unlike R, is not a pure circuit property, as it depends upon frequency.

To illustrate the foregoing we may consider a resistance-free inductive circuit of $\frac{1}{2}$ henry. The reactance ($\pi=3{\cdot}14$) will be $2\times3{\cdot}14\times50\times\frac{1}{2}=157$ for a 50 cycle supply. The current in the circuit with a pressure of 230 volts will be $230/157=1{\cdot}47$ amperes. Reactances are reckoned in ohms.

Capacitative Circuits.

There is another kind of circuit that takes current from an A.C. supply while absorbing practically no power, and this is one that comprises an electrical condenser. A condenser consists of two metallic sheets or plates separated by a thin zone of insulating material. If these plates are connected to a source of steady voltage a momentary current will flow, after which the condenser behaves as an open circuit. The transient flow of current is called a charge, and it is measured as a quantity of electricity in ampere-seconds. The charge is proportional to the applied pressure, and the ampere-second charge per volt is called the capacitance in farads.

If a condenser is connected to a supply giving an alternating pressure, the charge in it will follow the pressure variations and will therefore vary continuously. To alter the charge continuously current must flow in and out of the condenser. At any instant this current in amperes will evidently be equal to the rate of change of the charge in ampere-seconds a second.

Consider fig. 5. Here the full line curve represents the variations of pressure and quantity. At the point A the charge is increasing at its greatest rate and current is therefore passing into the condenser in the same direction as the applied

pressure. At B the quantity in the condenser is momentarily stationary and the current is then zero. At C electricity is passing out of the condenser at the maximum rate, and the current is therefore a maximum in the direction reverse to what it was at A. It is therefore clear that the maximum values of the current occur $\frac{1}{4}$ cycle

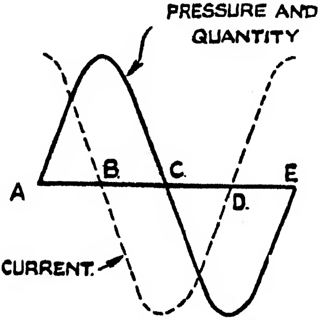

FIG. 5.

earlier than the corresponding maxima of the pressure. In other words the alternating current in a condenser leads the applied voltage in phase by $\frac{1}{4}$ cycle or 90°. This current produces negligible heating because the condenser plates need have no more than negligible resistance. The power must therefore be zero. The power factor is also zero. Zero power factor applicable to an inductance is evidently not quite the same as that applicable to a condenser. The one is said to be lagging, the other leading.

The A.C. in a condenser per R.M.S. volt of pressure can easily be calculated on the assumption that the current is sinusoidal. If the farad capacitance is C, the maximum charge per R.M.S. volt is $\sqrt{2}C$, and in the interval B to D (fig. 5) the charge changes by $2\sqrt{2}C$ in $\frac{1}{2}$ cycle or in $1/2f$ seconds. The average current is therefore $4\sqrt{2}fC$ amperes, and the R.M.S. value, being average value multiplied by the form factor $\pi/2\sqrt{2}$, is $2\pi fC$. The reciprocal of this quantity, $1/2\pi fC$, is called the capacity reactance X, for the current I at a pressure V being $V \times 2\pi fC$ is equal to V/X, and X is like the resistance R in Ohm's law $I = V/R$.

This result may be illustrated by a numerical example. Commercial capacitances are reckoned in microfarads (μF) or millionths of a farad, so we shall calculate the current in a 100 μF condenser when connected to a 50 cycle, 230 volt supply. $2\pi fC = 314 \times 100/1{,}000{,}000$ and $X = 1000/31{\cdot}4$. The current is V/X or $230 \div (1000/31{\cdot}4) = 7{\cdot}2$ amperes.

Current Resonance.

Let us now consider the double circuit of fig. 6. Here one branch A consists of a resistance-free inductance, the other, B, is a capacitance, and the two branches are connected in parallel to source of A.C. The current in each branch will be equal to

volts divided by reactance. Suppose the reactances are made numerically equal so that the currents are also equal. The one in A will lag 90°, and that in B will lead 90° on the supply pressure. The waves of pressure and of the two currents are indicated. It is seen that at any instant the current in B is equal and opposite in direction to that in A. The one current therefore completely

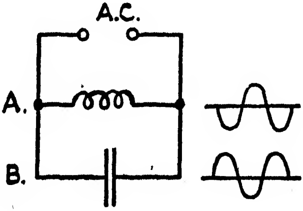

FIG. 6.

cancels the other and the current taken from the supply is zero. This curious phenomenon is called current resonance. It is clear that if the reactances and hence the component currents are unequal, the current from the supply will be equal to the difference of these component currents. If the inductance current is the greater, the resultant current from the supply will lag the voltage in phase by 90°. If the condenser current is the greater, the resultant will lead by 90°. Thus a wattless current leading by 90° can partially or wholly destroy a wattless current lagging by 90°.

Reactance with Resistance.

Fig. 7 illustrates a double circuit comprising a resistance-free inductance A and an inductance-free resistance B. If the reactance X of A is known the current in it from the A.C. supply of voltage V is easily determined. Let us consider in detail the current in the B branch, of resistance R ohms.

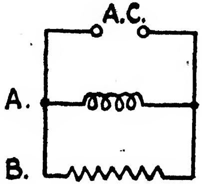

FIG. 7.

There will be no self-induced e.m.f. in this branch to oppose changes of this current. At any instant therefore the current will be given by Ohm's law, and the R.M.S. amperes will be equal to R.M.S. volts divided by R. Further maximum values of current and voltage will occur simultaneously. The phase difference between I and V will be zero.

The power loss in B branch is equal to the square of the R.M.S. current I multiplied by R. Now I^2R is equal to VI since $I = V/R$. Thus the power used in B, in watts, is equal to the volt amperes, so that the power factor is unity.

What will be the resultant current flowing from the supply into these two branches? The values of the components can be found, and the two currents are known to differ in phase by 90°. It is shown in text-books on A.C. theory that, provided the two component currents are each sinusoidal, the resultant current can be determined by the graphical construction of fig. 8. The lines OI_R

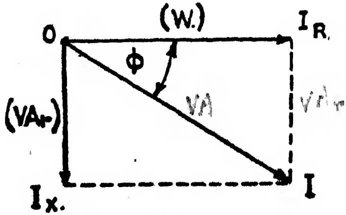

FIG. 8.

and OI_X are drawn at right angles to represent to scale the actual currents in the resistance and inductance branches respectively. The rectangle is completed and the diagonal OI gives to the same scale the magnitude of the resultant current from the supply. Moreover, the angle ϕ between OI and OI_R, represents the lag of the phase of this resultant on the supply pressure. The time lag expressed as a fraction of a cycle is the degree measure of ϕ divided by 360. The resultant current I can be calculated numerically as, evidently, $I^2 = I_R^2 + I_X^2$.

Now, as the power consumption W in B is equal

to the VA in it, the line OI_R also represents this power. Similarly OI represents the VA drawn from the supply and OI_X the volt amperes in the A branch, as all three currents are derived from the same pressure. The VA in the A branch being entirely wattless, and carrying no power are denoted by a special symbol VAr (voltamperes-reactive). We see now that the power factor of the supply to the double circuit, W/VA, is equal to the ratio of OI_R to OI in the diagram. This ratio is a characteristic function of the angle ϕ called its cosine, and is written $\cos \phi$. Knowing ϕ, $\cos \phi$ can be found from trigonometrical tables, and vice versa. The ratio of VAr to VA is called the sine of the angle ϕ, written $\sin \phi$, and the ratio of VAr to W is called the tangent of ϕ, written $\tan \phi$. These ratios are tabulated in the Appendix.

The VA per watt in an A.C. supply is evidently equal to $1/\cos \phi$, or $\sec \phi$, and increases as the power factor falls. The ratio $\tan \phi$ is equal to the VAr per watt, and also increases as the power factor falls. The lower the power factor, therefore, the greater the VA and the VAr per watt.

If the power factor of an A.C. supply is known, the angle ϕ of which this power factor is the cosine can be found from tables and thence $\tan \phi$ or the VAr per watt can be determined. Conversely knowing the VAr per watt or $\tan \phi$, ϕ, and $\cos \phi$ the power factor can be found from tables.

We now see that an A.C. supply has four characteristics: the voltamperes VA, the power in watts W, the reactive voltamperes VAr, and the angle ϕ of phase difference between the supply current and pressure. It should be carefully noted that, with sinusoidal currents, all these quantities are directly measurable, and that any two determine the remaining two. Thus knowing W and ϕ, $\cos\phi$ and $\tan\phi$ can be determined from tables, and $VA = W/\cos\phi$, and $VAr = W/\tan\phi$. Knowing W and VA we have $(VA)^2 = W^2 + (VAr)^2$ and $\cos\phi = W/VA$ whence ϕ can be found from tables. We shall deal fully in a subsequent chapter with the matter of these measurements, but it should now be understood that power factor can be calculated from any two of the quantities W, VA, VAr, or ϕ, with the important proviso that the current concerned is sinusoidal and is derived from a source of sinusoidal pressure. The importance of this proviso will be explained later.

Increase of Power Factor.

Fig. 9 illustrates a circuit in which a condenser branch C has been added to the combination shown in fig. 7. It should by now be clear that if the condenser current is less than the current in the inductive branch, the VAr drawn from the supply will be equal to the VAr in A, less that in C. The VA from the supply will be represented as

shown in fig. 10 by the diagonal of a rectangle of which the sides are W the power consumption in the resistance, and the net VAr taken from the

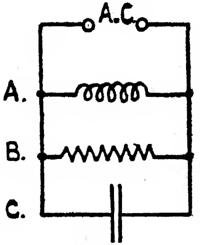

FIG. 9.

supply. This reduction in net VAr decreases ϕ and VA and increases the power factor. If the VAr in A and C branches are numerically equal, they will

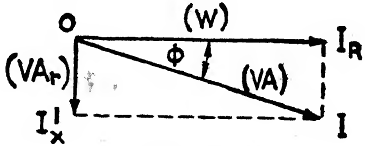

FIG. 10.

cancel; the net VAr taken from the supply will be zero, the supply VA will be equal to the watts and the power factor will be unity. If the condenser VAr exceed those in the inductance, the resultant current from the supply will lead in phase on the supply pressure, and the power factor of the supply will be less than unity and will be leading.

Watts, VA, and VAr.

Our study of the circuit shown in fig. 7 has led to the conclusion that two currents, one in phase with the supply pressure and one lagging this pressure by $\frac{1}{4}$ cycle will combine to form a third current lagging the pressure by an amount depending upon the watts carried by the one current and the VAr carried by the other. In this circuit the watts and VAr exist objectively in the two separate branches, but it will be clear, on reflection, that the possibility of considering the total VA output of a supply to be composed of a watt and a VAr component does not depend upon the nature of the circuit in which this supply is absorbed. Whatever the nature of this circuit may be, if the power factor of the input to it is between unity and zero, this input may be considered to contain a watt and a VAr component, because it is possible to construct a circuit like fig. 7, in which these two components appear objectively in separate branches.

There are many circuits taking an A.C. supply at a power factor between unity and zero. A very simple circuit of this kind is shown in fig. 11. This consists of a resistance-free inductance X and an inductance-free resistance R in series. If X and R represent respectively the reactance and resistance of these components in ohms, then it is shown in the text-books that, with sinusoidal current and

voltage the power factor of the supply is equal to $R/\sqrt{(R^2+X^2)}$. The quantity $\sqrt{(R^2+X^2)}$ is called the impedance, and is denoted by the symbol Z. The current is V/Z.

In most circuits inductance and resistance are not separate as in fig. 11, but are combined, so that the whole circuit possesses both resistance and

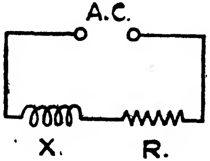

FIG. 11.

inductance. In this case the circuit behaves as if these two characteristics were separated as they are shown in fig. 11, and the current and power factor are respectively equal to V/Z and R/Z.

In the circuit of an A.C. motor, the majority of the electrical power is not converted into heat but into mechanical power. In every kind of circuit, however, the input from the supply is equivalent to a watt or power component flowing simultaneously with a wattless or VAr component, if the power factor is less than unity.

Numerical Examples.

We may illustrate this very important principle by some simple numerical examples.

(1) The input to an A.C. load is 2 kW at a measured power factor of 0·8. Find the kVA and the kVAr. kVA = kW/power factor $= 2/0{\cdot}8 = 2{\cdot}5$ kVA. From tables the value of $\tan\phi$ corresponding to $\cos\phi = 0{\cdot}8$ is found to be 0·75, whence $\text{kVAr} = 2 \times 0{\cdot}75 = 1{\cdot}5$. Otherwise as $(\text{kVAr})^2 = (\text{kVA})^2 - (\text{W})^2 = 6{\cdot}25 - 4 = 2{\cdot}25$ the $\text{kVAr} = \sqrt{2{\cdot}25} = 1{\cdot}5$.

(2) An A.C. load supplied at 230 volts takes 2 amperes and 300 watts. Find the power factor and the VAr. $\text{VA} = 2 \times 230 = 460$. Power factor $= \text{W/VA} = 300/460 = 0{\cdot}65$. $(\text{VAr})^2 = 460^2 - 300^2 = 211600 - 90000 = 121600$, and $\text{VAr} = 334$.

(3) The measured kW and kVAr input to an A.C. load are respectively 2 and 1. Find the kVA and the power factor. $(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAr})^2 = 4 + 1 = 5$. $\text{kVA} = \sqrt{5} = 2{\cdot}24$. Power factor $= \text{W/VA} = 2/2{\cdot}24 = 0{\cdot}895$.

Distorted Waves.

We have been very careful to stipulate that the principles explained in detail in the preceding sections are true only if current and pressure waves are sinusoidal. This limitation does not greatly impair the practical utility of these principles because the stipulation is very approximately satisfied in most A.C. supplies, as those for heating, lighting and power. There are, however, certain supplies that take a distorted wave of current with a sinusoidal

pressure. The most important of these are the supplies to vacuum discharge lamps. A discharge lamp has no reactance, but its resistance varies with the current it carries. Consequently, as the voltage varies from zero to maximum, the resultant current does not vary proportionally, and the shape of the current wave differs from that of the pressure. Another example of a supply with a distorted current wave is that to a transformer supplying no secondary load. This supply is merely used to magnetise the core. The reactance of the magnetising circuit is not constant, because successive increments of magnetism of the iron core require greater and greater increments of magnetising current. A third example of a distorted current wave is the input to a mercury arc rectifier supplying direct current. This distortion arises because the current from the A.C. supply flows intermittently.

In all circuits, whatever the wave forms of current and pressure may be, the fundamental definition of power factor given on p. 7 applies. Power factor is the ratio of power consumption in watts to the product of R.M.S. volts by R.M.S. amperes. This is the only correct definition of power factor. The statement found in many books that power factor $=\cos\phi$ is not a definition because, as we shall see, it is not always true.

If the input to a transformer supplying no

secondary load is measured in watts, amperes, volts, and VAr, the power factor calculated from W and VA, the actual value, will be different from and less than the power factor calculated from W and VAr. In this case $(VA)^2$ is not equal to but greater than $W^2 + (VAr)^2$. We can represent this discrepancy algebraically by the equation $(VA)^2 = W^2 + (VAr)^2 + D^2$ where D^2 represents the discrepancy. This anomalous quantity D^2 is due entirely to the distorted shape of the current wave. Till now we have considered power factors of less than unity to be due to either leading or lagging VAr. It is easy to see however that even if we could cancel the VAr in the supply to the transformer, we should still have $(VA)^2 = W^2 + D^2$, because we should not have corrected the wave distortion. Thus, while this distortion is present VA must necessarily always be greater than W and the power factor must be less than unity.

The following illustration, due to Mr H. Rissik, may help to elucidate this rather difficult point. Consider the circuit shown in fig. 12. Here an alternator A supplies a circuit consisting of an inductance-free resistance R of 10 ohms, and a battery B giving a steady voltage of 50. The R.M.S. voltage given by the alternator is 100. Consider that the resistances of A and B are negligible. Now both A and B will circulate current through the circuit. A will give an A.C.

of 100/10 = 10 R.M.S. amperes, and B will give a direct current of 50/10 = 5 steady amperes. The

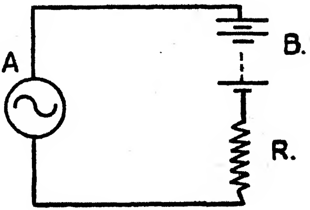

FIG. 12.

actual current flowing from the alternator, being compounded of these two currents, will have a wave-form like that shown in fig. 13. Although

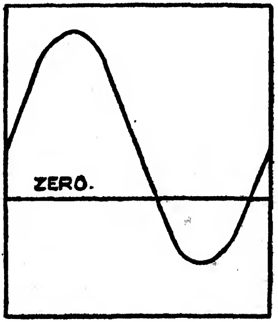

FIG. 13.

the shape of the curve is like that in fig. 3, it is not sinusoidal, because it is not symmetrical about the zero line. The superposition of the D.C. on the A.C. increases the R.M.S. value of the actual

current coming from the alternator and it can be shown that this R.M.S. value is equal to $\sqrt{10^2+5^2}=11{\cdot}18$ amperes. Now consider the actual power output of A. As R is free from inductance, the A.C. component of the current will be in phase with the pressure of A and this will give power equal to $10\times100=1000$ W. The D.C. combining with the alternating pressure of A gives no *average* power because instantaneous values of this power will be in reverse directions during successive half cycles. Thus, while the VA output of A is $11{\cdot}18\times100=1118$, the power output measured by a wattmeter would be 1000 only. The power factor therefore is $1000/1118=0{\cdot}895$. Now, in this circuit there can be no VAr because R is inductance-free, and if the VAr output of A were measured, the instrument would give zero reading. The power factor of 0·895 is therefore the maximum possible, and it cannot be increased.

We may here remark, that the fictitious power factor of a circuit with a distorted current wave, calculated from W and VAr or as the cosine of an angle is called the displacement factor. The true power factor is less than $\cos\phi$ and may be symbolically expressed as equal to $\mu\times\cos\phi$ where μ, less than 1, is called the distortion factor. Thus, in the circuit of fig. 12, the displacement factor, $\cos\phi=1$, and the distortion factor, $\mu=0{\cdot}895$. The power factor, $\mu\times\cos\phi$ is 0·895. (See Appendix.)

3-Phase Power Factor.

Supplies of electrical energy by public authorities are now usually given on the 3-phase A.C. system by four conducting mains comprising three "lines" and a "neutral." Two sets of voltages are obtainable from such a system, the phase voltage between any line and neutral, and the line voltage between any two lines. The line voltage is equal to $\sqrt{3}$ times the phase voltage. Power can be taken from the system in two ways; the first, for lighting and heating, by two conductors connected to a line and the neutral or, rarely, to two lines; the second, for power purposes by three conductors supplying three interconnected circuits in a motor. With any combination of these kinds of supply, the whole input to the 3-phase supply system constitutes what is called a 3-phase load, and this may be considered as the resultant of three component loads, each of which is delivered through a line and the neutral, this neutral acting as a "common return" for all three lines.

If all three line currents are equal, and have mutual phase differences of $\frac{1}{3}$ cycle like the phase voltages, the lags of line currents on corresponding line voltages will be the same. The power factors of all component loads will be equal, and this power factor may be called the power factor of the whole 3-phase load. Such a load is said to be

balanced. If however the component power factors are unequal, there is no obvious way of assigning a power factor to the whole 3-phase load.

One possible and apparently rational method is to obtain the total power carried by the 3-phase load and to add the three values of VA obtained by multiplying each line current in amperes by the

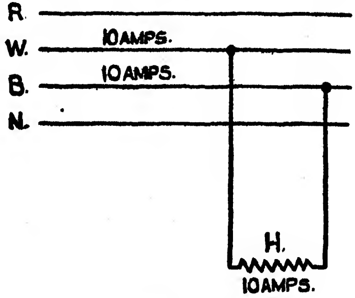

FIG. 14.

phase voltage. The ~~rates~~ ratio of total watts to total VA so obtained gives a value of 3-phase power factor.

It is easy to show that this method can lead to an anomalous result. Consider fig. 14, which shows an inductance-free resistance, say an electric heater, H, connected between two lines of a 3-phase system, and taking 10 amperes therefrom. We assume that the voltages have the standard value for ordinary distribution, *i.e.* line voltage 400, phase voltage 230. The actual supply used in H is of unity power factor, and both W and VA are

$10 \times 400 = 4000$. If this heater is the only consuming device connected to the four supply mains, the power input to these mains is 4000 watts. If we obtain the total VA in these mains we find the 10 amperes in those marked W and B are each associated with a phase voltage of 230. The total VA value is thus $2 \times 230 \times 10 = 4600$, and the ratio of watts to total VA to $4000/4600 = 0{\cdot}87$. Now a definition of 3-phase power factor that assigns a value of 0·87 to a supply absorbed in an inductance-free resistance is plainly unsatisfactory, because it implies that this supply includes VAr which as a matter of fact it does not. The anomaly here arises from the fact that, whereas each line carries half the power, 2000 watts, it also carries VAr because the VA are 2300. The VAr in one line however lead in phase while those in the other lag in phase similarly, and the two line VAr cancel and do not appear in the actual consuming device. The conditions illustrated in fig. 14 represent an extreme case, because a 3-phase set of mains would, in practice, hardly ever be required to supply a single heater only. As, however, a satisfactory definition must cover extreme as well as ordinary conditions, we must conclude that to reckon 3-phase power factor as the ratio of total watts to total VA is not quite satisfactory.

Before explaining the conventional definition of 3-phase power factor we may note, what will be

dealt with more fully hereafter, that the whole power in a 3-phase supply can be measured by a single instrument. Moreover a single instrument will measure the total VAr, not in the lines considered *in reference to* phase voltages, but actually appearing in consuming devices. Such an instrument, for instance, when measuring VAr in the 3-phase system of conductors of fig. 14 would give zero reading because no VAr are taken in H. The square root of the sum of the squares of the total watts and the total VAr so defined gives a quantity called the total equivalent volt-amperes; or representing this last quantity by VA, we have $(VA)^2 = W^2 + (VAr)^2$. The ratio W/(VA) is called the 3-phase power factor. The meaning of total equivalent voltamperes is that it is equal to the actual total VA in a balanced load carrying the same power W at the same 3-phase power factor. The total equivalent voltamperes in a 3-phase load is equal to the actual total VA if the load is balanced, otherwise it is always less (see Appendix). The difference between these two values of the 3-phase VA is generally small in practice, and would rarely exceed 1 or 2 per cent. The values of the 3-phase power factor calculated from these two values of the VA will likewise be very nearly the same in practice. Even small differences caused by differences of definition may however be of practical importance when charges

for a 3-phase supply are made to depend upon 3-phase power factor.

The standard definition of the power factor of a 3-phase supply is, then, given by the formula—

$$\text{Power factor} = \frac{W}{\sqrt{\{W^2 + (VAr)^2\}}},$$

where W and VAr represent total values, referred to consuming devices. The importance of this definition arises from the fact that nearly all practical methods of measuring 3-phase power factor lead to this value.

It may be noted, in conclusion, that 3-phase power factor, never being greater than unity, is always equal to the cosine of some angle. This angle however does not represent any actual phase difference of current and voltage in the supply excepting when the 3-phase load is balanced, or in an extreme case like that illustrated in fig. 14, when only one consuming device is connected to the supply system.

Average Power Factor.

Power factor, like watts and amperes, is inherently a property of an electric supply that is of a variable character. Variation of power factor may arise from an alteration of the character of the consuming devices used or from changes in the mechanical loads imposed on A.C. motors. When

the power factor of a supply varies it is often desirable to assign some value to the average power factor over a period.

As a general rule it may be said that the more complex the nature of a variable quantity, the more difficult it is to define what is meant by its average value. Power factor is a complex quantity involving volts, amperes, and watts, and we shall see that it is not easy to say exactly what is meant by the average when this quantity varies.

Suppose, for instance, that over a period of 8 hours a consumer's load is 10 kW at 0·9 power factor constant for the first 4 hours, and 2 kW at 0·3 power factor constant for the remainder of the period. There is a sense in which average power factor is the mean of the two values 0·9 and 0·3, or 0·6, but as the time of low power factor is associated with a relatively small load, this value is manifestly not quite satisfactory.

Let us consider a more simple average, that of average power supply over a period. It is evident at once that this average is easily obtained if the energy consumption, as given by a meter, is known. Average power in kW is energy consumption in units or kWh, divided by time in hours. This average is definite and unequivocal.

A meter registering units or kWh is said to integrate the power because, over successive very short periods it measures and adds up the product

of average power by time. Special meters can be made similarly to integrate the quantities kVA and kVAr, and the registrations of these meters give kilovoltampere hours (kVAh) and kilovoltampere hours reactive (kVArh).

Suppose that a supply is metered in kWh and kVAh, then dividing registrations over the same period by the duration of this period in hours gives average kW and average kVA and the ratio of these averages gives a rational kind of value of average power factor. This average power factor is evidently equal to the ratio kWh/kVAh. The supply can also be metered in kWh and kVArh, and these meter registrations represent corresponding averages of kW and kVAr. This evidently leads to another value of average power factor defined by the formula

$$\frac{\text{kWh}}{\sqrt{\{(\text{kWh})^2 + (\text{kVArh})^2\}}}.$$

We see, then, that there are at least three possible definitions of average power factor; the first a simple time average, the second depending on kWh and kVAh, and the third on kWh and kVArh. Let us see, by a simple numerical working, whether these three definitions lead to concordant values. In the following table the first three columns show the supposed constant characteristics of an A.C. supply over three time periods. The next three columns give the advances in the

registration of kWh, kVAh, and kVArh meters over these periods.

Time in Hours.	kW.	Power Factor.	kWh.	kVAh =kWh/Power Factor.	kVArh $=\sqrt{\{(\text{kVAh})^2 - (\text{kWh})^2\}}$.
1	2	0·5	2	4	3·46
1	4	0·8	4	5	3
1	10	1·0	10	10	0
Total meter advances			16	19	~~8~~·46 6

The time average of the power factor is $\frac{1}{3}(0{\cdot}5+0{\cdot}8+1{\cdot}0)=0{\cdot}77$. The average value based on kWh and kVAh is $16/19=0{\cdot}845$. The average value based on kWh and kVArh is

$$\frac{16}{\sqrt{(16^2+8{\cdot}46^2)}}=\frac{16}{\sqrt{(256+72)}}=\frac{16}{18{\cdot}1}=0{\cdot}885.$$

We see at once that the three average values are very discordant, the time average is the least, and the value based on kWh and kVArh is the greatest. This is generally the case, excepting when the power factor is constant, although the discordance of the three values in actual practice would usually be less than that found in our manufactured example which, although quite possible, is merely used for the purpose of illustration. (See Appendix.)

The question arises then: which is the correct definition of average power factor? The answer

to this question is that there is no correct definition. As with 3-phase power factor so with average power factor, we must choose a definition which is convenient and adopt it as a convention. So far no definition of average power factor has been conventionalised authoritatively, but it may be noted that as kVArh is, after kWh, the quantity that can be the most conveniently and the most accurately measured, the definition based on this quantity is, on practical grounds, the best. We note also the important fact that the average power factor obtained from kWh and kVArh is always greater than the value obtained from kWh and kVAh, excepting when the power factor does not vary in which case, of course, the two values are the same.

CHAPTER II

MEASUREMENTS

Methods of Measuring Power Factor.

We have seen that power factor is a characteristic of an A.C. supply which depends upon power, current, pressure, VAr, and phase difference of current and pressure. Power, current, and voltage determine power factor, irrespective of wave form. VAr and phase difference only determine power factor when wave forms are sinusoidal.

Two kinds of measurement of power factor are possible. The first kind depends on the readings of one or more pointer indicating instruments. Methods of this kind give the value of power factor at the instant the readings were taken. Methods of the second kind depend upon observation of the advances of integrating instruments of the meter type over the same period. Power factor determined in this way is an average value, but, if the averaging period is short, and power factor variations are small during this period, such a value is generally as useful as one obtained by pointer instruments.

The subject of the measurement of the electrical

quantities involved in the determination of power factor forms a specialised technique. In this chapter we shall explain these methods briefly and simply, with a view to giving those practically interested in power factor problems a general kind of idea of the principles underlying them.

Power Factor from Power, Current, and Voltage.

This is the most obvious and general method of measuring power factor. Since it involves the quantities appearing in the fundamental power factor definition it is theoretically accurate irrespective of wave forms. There are however some practical drawbacks to this method in circumstances where wave forms are known to be very approximately sinusoidal and other methods are therefore legitimate. In the first place three instruments have to be read simultaneously: a difficult matter if the current and power are varying. Secondly, as no instrument is perfectly accurate, there are three sources of error, and the three instrument errors may combine to give a considerable error in the final value of the calculated power factor. If however the switch-control equipment for a consuming circuit contains the instruments necessary for this method it is a very convenient one, for a power factor value can be calculated at any time and without special preparation. Even if a watt-

meter and an ammeter only are provided an approximate value of the power factor can be computed from the readings of these instruments by assuming that the voltage has its nominal or declared value. If for instance the power input to a 400/230 volt 3-phase circuit is 20 kW and a line current is 50 amperes, then, assuming the load is balanced, the kVA will be $3 \times 50 \times$ (the phase voltage), or $3 \times 50 \times 230 = 34{\cdot}5$, and the power factor is 0·58.

The average power in kW over a short period of a few minutes can be obtained if a meter is available instead of a wattmeter. All meters are provided with small or testing dials reading in fractions of a kWh, and the advance of the pointer over the dial of the smallest denomination can usually be determined with fair accuracy over a five-minute period. Twelve times this advance will evidently give the units an hour or the average kW, and if the reading of the ammeter has been so steady over the period that its average value can be estimated, a useful approximate value of the power factor can be obtained.

The average power in a circuit provided with a kWh meter can be obtained over a still smaller period by timing a few revolutions of the meter disc by means of a stop watch. On the dial plate or the number plate of the meter will be found its characteristic gearing constant "revolutions per

unit" (R.P.U.). The average power in kW is obtained from the formula

$$\mathrm{kW} = \frac{\mathrm{revs} \times 3600}{\mathrm{secs} \times (\mathrm{R.P.U.})}$$

Direct Indication of Power Factor.

An instrument that indicates the value of power factor directly is called a power factor meter. The

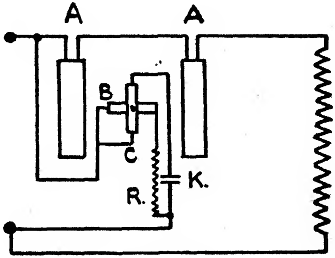

Fig. 15.

action of a power factor meter for circuits with two conductors will be understood in a general way by a brief study of fig. 15. AA are two fixed circular coils with their axes in line, and these coils carry the current of the circuit in which the power factor is required. The moving system consists of two small circular coils, B and C, set at right angles in line with AA and pivoted so that they can turn freely, without any spring control. One coil B is joined to the supply terminals by fine conducting ligaments through a resistance R, and the other,

C, through a condenser K. Suppose that the current in AA is in phase with the voltage, and hence with the current in B, the mutual action of the currents in AA and B will be like that in a wattmeter and B will tend to set its axis in line with that of AA. The current in C leads nearly 90° in phase on the current in AA, and the mutual action of the currents in AA and C will be like that in a wattmeter measuring power at zero power factor, and will be nil. The moving system will therefore set itself so that coils AA and B are in line, and the pointer will indicate 1 on the scale. If the current in AA leads 90° on the voltage, the mutual action of coils AA and B will be nil and coil C will set itself in line with coils AA and the pointer will indicate 0 leading. If the current lags 90°, then, as this is a reversal relative to a 90° lead, the coil C will set itself in line with AA, but in the opposite direction, and the pointer will stand at a point opposite to 0 leading and will indicate 0 lagging. In short, the mutual action of the currents in coils AA and B depends upon the watts, that of the currents in coils AA and C depends upon the VAr. At power factors intermediate between 1 and 0, both sets of coils will control the position of the moving system and its position will depend upon the ratio of VAr to W. Thus the scale of the instrument can be marked so that the pointer indicates power factor values.

Three-phase power factor meters for balanced loads have either one current and three voltage coils or one voltage and three current coils. The sets of three coils are physically arranged, so that they are at angles with each other corresponding to the phase differences of the currents in them, *i.e.* at 120 degrees. Three-phase power factor meters for unbalanced loads have three current and three voltage circuits, and indicate a power factor value based on the rates of watts to total equivalent voltamperes explained on page 33.

If a power factor meter is connected in a circuit, and the current in this circuit is not sinusoidal, the instrument will indicate the displacement factor which as has been explained on p. 28 is greater than the actual power factor.

Power Factor Measurement by A.C. Milli-Ammeter.

An interesting and useful method of directly indicating approximate power factor values depends upon the superposition of two currents in a single ammeter designed primarily to measure small alternating currents. Fig. 16 is a diagram of connections for measurements of this kind. A is the instrument which is joined to the supply terminals through a variable resistance R. A is also joined to the secondary winding of a current transformer CT with a U-shaped core that can be

slipped round a conductor carrying A.C. To measure power factor, the resistance R is adjusted so that the pointer indicates half full scale value. With the current in this resistance still passing, the core of CT is brought near to a conductor carrying current of the supply of which the power factor is required. In this position the transformer

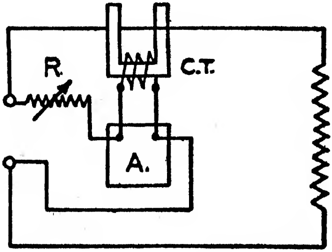

FIG. 16.

will supply a second current to A, the value of which will depend, not only on the value of the supply current, but also on the position of CT relative to the conductor, and this second current will modify the resultant current in A upon which the reading depends. The position of CT is adjusted till the reading of A is what it was before CT was put near the current-carrying conductor, *i.e.* half scale value. With the transformer kept in this position, the current from the supply terminals through R is interrupted by means of a switch. The reading of A now depends

upon the current from CT only, and this reading indicates the power factor on a special scale provided. The principle underlying this method of measurement is that the current from CT required to give a resultant in A of a value equal to the current in R depends upon the phase differences of load current and pressure and therefore on the power factor.

An alternative method of using the instrument for the measurement of the power factor of balanced 3-phase loads is as follows. The current in the circuit containing R is derived from two lines of the supply and is adjusted so that the pointer indicates full-scale value. The core of CT is now brought near to the third line conductor and a position is found for which the reading of A is the least possible. In this position the pointer of A indicates power factor on the power factor scale.

Power and Energy Measurements.

We have already explained in Chapter I that the power in any kind of load supplied by two conductors can be measured by a single instrument called a wattmeter, and that the energy consumption by the load can be measured by an integrating instrument generally called a meter.

The total power in a 3-phase 3-wire load supplied by three conductors can be measured by two wattmeters connected as shown in fig. 17. The

legitimacy of the method depends upon the fact that any kind of 3-phase 3-wire load can be artificially reproduced by two suitable consuming devices connected between two pairs of lines. The readings of two wattmeters connected as in fig. 17 will depend, not only on the total power but also on the balance of the load and upon the 3-phase power factor. The total power is equal to what

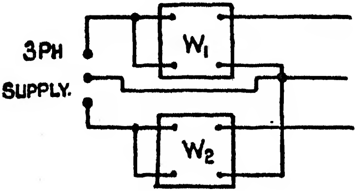

FIG. 17.

is called the algebraic sum of the wattmeter readings; this means that if one of the instruments, being correctly connected, gives a reverse reading, the value of this reverse reading must be subtracted from that of the second instrument to obtain the total power. If a wattmeter gives a reverse reading, it is often made to read forward by changing the direction of the current in one of its coils, or by artificially making the connections incorrect.

In this method of power measurement it is important to be sure that the two wattmeters are correctly connected, so that a true reverse reading is recognised. Provided the general method of

connection corresponds to fig. 17, the correctness of the connections can be inferred from two simple rules: (*a*) the instrument giving the greater reading should read forward, (*b*) the second instrument should always read forward when the connection to the middle line is removed and the two wattmeter voltage circuits are left in series across the other two lines.

Three-phase wattmeters are available which contain two separate instrument elements and one spindle. The two elements are connected as in fig. 17, and the pointer indicates total power.

The total power in a 3-phase supply given by four conductors, is measured by three wattmeters, each carrying a line current and having its second circuit connected between this line and the neutral.

The energy consumption of a 3-phase 3-wire load can be measured by two meters connected as in fig. 17, or, as is generally the case, by a single instrument with two elements, which gives the total consumption on one dial. Three-phase 4-wire meters contain three elements and register the total energy consumption in a 3-phase supply by four conductors.

Approximate 3-Phase Measurements.

If a 3-phase load is balanced then, as we have seen on page 29, the total power is three times the component power carried by each line, and the

value of this power can be determined by a wattmeter carrying a line current and having its second circuit connected to this line and the neutral. If, as is often the case, the neutral conductor is earthed near the measuring point, then, for temporary measurements, the wattmeter circuit may be connected between a line and earth.

VAr Measurements.

The correct indication of a wattmeter depends upon one of its circuits carrying the current of the load measured and the second or voltage circuit being connected to the pressure at which this load is supplied. The reading of the instrument then gives the active or power component of the VA in the load. If, by some artificial means the voltage circuit is supplied by a pressure equal to the actual supply pressure, but displaced in phase therefrom by 90 degrees, the reading of the instrument will no longer give the active or power component of the VA, but the wattless or VAr component. Methods of measuring VAr or registering kVArh nearly all depend on this basic principle. The indication of VAr, although presenting no technical difficulties, is in practice required much less than the registration of kVArh, by integrating instruments generally called reactive meters and sometimes "sine" meters. This second designation is misleading and should be avoided.

Reactive meters in commercial use are generally of the 3-phase 3-wire kind; they contain two elements each of which carries a line current and is supplied in its voltage circuit from a pressure lagging 90 degrees in phase on the line pressure that would be required for the registration of energy. The actual registration of kVAr by a meter of this kind corresponds to the value of 3-phase kVArh described on page 32, and the rate of registration depends on the kVAr consumption in the load, and not on the apparent kVAr in the three lines. The artificial pressures for a reactive meter are derived from the 3-phase circuit itself. That this is possible can be explained by the fact that as there is a 120 degree phase difference between line voltages, and a 30 degree phase difference between a line voltage and a corresponding line-to-neutral voltage, it is possible to find an artificial voltage displaced 120 minus 30, or 90 degrees out of phase with the one required for power or energy measurements.

Approximate VAr Measurement.

There are two important practical methods of measuring VAr in 3-phase circuits supplying balanced loads. The first is an application of what is often called the two-wattmeter method of measuring 3-phase power which was described on page 46. The VAr can be determined at once

from the rule that if the 3-phase load is balanced the VAr are equal to $\sqrt{3}$ times the algebraic difference of the wattmeter readings. Algebraic difference means that if the wattmeter readings are added to obtain power in watts, the lesser is subtracted from the greater to obtain VAr; if the lesser is subtracted from the greater to obtain watts, the readings are added to obtain VAr.

It is important to remember that, whereas the algebraic sum of the wattmeters always gives the total watts in the supply, the VAr are only equal to the algebraic difference of the readings when the load is balanced.

When two meters are used to measure 3-phase energy, the algebraic difference of their registrations gives the kVArh consumption provided the load is always balanced throughout the registration period.

A second simple method of obtaining the VAr in a balanced 3-phase load is by connecting a wattmeter as shown in fig. 18 so that it carries the current in one line and has its voltage circuit joined to the other two. A meter can be connected similarly to register kVAr. The 3-phase VAr is equal to $\sqrt{3}$ times the wattmeter reading, or the kVArh consumption to $\sqrt{3}$ times the meter registration. It will be noted that in this method of measurement the instrument must be suitable for

the line voltage. A wattmeter suitable for this voltage can be used for power measurement by the method described on page 48, but a wattmeter or meter designed for the phase voltage cannot be used with its voltage circuit supplied at line pressure.

The foregoing methods of approximate VAr measurement may be considerably in error if the

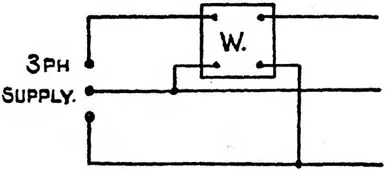

FIG. 18.

3-phase load is not balanced. A 5 per cent. difference in the values of two line currents may lead to an error of the same order. The supply to 3-phase motors is usually sufficiently balanced for these methods of VAr and kVArh measurements to yield results that are accurate enough for many practical purposes. In circumstances where meter readings of kWh are used as a basis for assessing charges for the supply, approximate methods of measurement are not allowable, and it is generally considered good practice to avoid approximate methods of measuring kVArh in these circumstances.

Maximum kVA Demand.

The most important value of the volt-amperes in an A.C. supply, and the one that is most frequently required in commercial measurements is the maximum average over a short definite period. Such a value is generally called the maximum VA or kVA demand, and, as we shall see later, it is used as a basis for assessing the charge for the supply under tariffs designed to make the overall unit rate depend upon the power factor. A maximum average value of kVA can be obtained in two ways. The best is by means of a special attachment to a meter registering kVAh called a demand indicator. A demand indicator is a mechanism whereby a pointer is pushed forward over a scale by gearing which engages mechanically with the movement of a kVAh meter. The engagement of the indicator gearing lasts for specified equal periods which may be 15, 20, or 30 minutes, and which may be called averaging periods. At the end of the averaging period the indicator gear is momentarily taken out of engagement, when the gearing is returned by a spring to the initial position. The demand indicator pointer however not being rigidly attached to the gearing remains at the point to which it was pushed over the scale. If in any subsequent averaging period, the rotation of the meter, and hence, the kVAh

consumption, exceeds that corresponding to the indicator reading the pointer will be pushed further over the scale, but not otherwise. The position of the pointer therefore corresponds to the maximum kVAh consumption in any averaging period. As all these periods are the same, the reading is a measure of the maximum average kVA, since this average is equal to maximum kVAh consumption divided by the averaging period in hours. The scale of the indicator is marked in average kVA. The temporary disengagement of the indicator gearing at the ends of the averaging periods is controlled by a clock. A demand indicator of this kind, which of course may be used to indicate maximum kW as well as kVA is usually known as one of the Merz type.

Instead of using a demand indicator in conjunction with a kVAh meter, the rotating element may be operated by current. The reading of the indicator will then correspond to maximum average current, but, if the supply pressure is constant, the maximum will be a measure of maximum kVA. The Hill-Shotter kVA demand indicator is constructed on this principle, and it can be designed to correct for small variations of the pressure. As the speed of the rotating part of an instrument of this type cannot be made proportional to the current operating it, the scale of the demand indicator is not uniform, as is the scale of an

indicator responsive to the registration of a true kVAh meter.

A second method of measuring maximum kVA demand is by the use of an instrument like an ammeter excepting that the correct indication of the current in it is only given after this current has been flowing for a stipulated period, which may be 15, 20, or 30 minutes. The indication of an instrument of this type depends upon the heating effect of the current, and the time lag of the true indication is due to the fact that a considerable amount of heat has to be generated before the final temperature, which determines the pointer position, is reached. The actual indication of an instrument of this kind is in kVA corresponding to the nominal pressure of the supply, and demand indicators depending upon this method of operation are usually designated as of the thermal type.

Two demand indicators, the one of the Merz type operated by a kVAh meter, and the other of the thermal type, will, if correctly calibrated and if the pressure is constant at the nominally correct value, indicate the same values of a demand if this demand is constant throughout the same averaging periods of the two instruments. If however the demand during averaging period contains a short peak value, the thermal indicator will tend to give the higher reading. This is because, with constant kVA, and with the pointers of the two instruments

starting from zero, the reading of the Merz indicator will increase at a constant rate, whereas the reading of the thermal indicator increases at a rate that is initially greater than, but finally less than the corresponding rate of the Merz indicator. A short peak lasting only a fractional part of the averaging period may therefore give a greater pointer advance in the thermal indicator.

kVAh Meters.

The simplest kind of kVAh meter consists of a direct current electrolytic meter which is supplied with rectified A.C. from a transformer in the supply circuit. The meter current being proportional to the supply current, the meter can be made to register kVAh at the normal pressure of the supply. An instrument of this kind is suitable only for 2-wire supplies, and, as kVAh measurements are generally required for 3-phase supplies, it is not of great practical importance.

There are several types of kVAh meters for 3-phase supplies. The technical principles underlying the action of these meters are somewhat complicated, and, in a book of this character, they can be described only in a general kind of way.

The simplest type of 3-phase kVA meter is an ordinary kWh meter compensated internally so that it will register kVAh at a stipulated power factor. If the actual 3-phase power factor differs

from this value, the meter will under-register, but small variations of the power factor will not cause errors of more than 2 per cent. An inherently inaccurate meter is not, however, very suitable for the measurement of important supplies, and consequently, this type of instrument is not used now to any extent.

The "Trivector," and the Westinghouse R.I. kVAh instruments each contain a 3-phase kWh and a reactive 3-phase kVArh meter. The pointer of the kVAh register and of the kVA demand indicator is, by somewhat complicated mechanical gearing, caused to rotate at a speed that is equal to the square root of the sum of the squares of the speeds of the kWh and the kVArh meter components.

Another type of kVAh instrument, recently developed by Casson & Gray, comprises a single 3 phase meter, the voltage supply of which is controlled by a power factor relay in such a way that the apparent power factor at which the meter measures is always approximately unity. The speed of registration of an ordinary 3-phase kWh meter is proportional to watts which is equal to 3-phase VA multiplied by power factor. In the kVAh meter the speed of registration is equal to 3-phase VA multiplied by the approximately constant artificial power factor of unity, and, hence, the meter registers kVAh.

kVAh meters of each of the three types just described all register at a speed corresponding to the quantity "total equivalent volt-amperes" which has been defined on page 32.

The Hill-Shotter 3-phase kVA demand indicator contains three current operated elements, which were described in the preceding section, and these elements act on a single moving system to which the pointer of the indicator is geared. The 3-phase kVA value indicated by this instrument does not correspond to "total equivalent volt-amperes," but to a quantity depending on the squares of the values of the three-line currents. Theoretically, with unbalanced 3-phase loads, the indication of a Hill-Shotter instrument will not only be greater than the "total equivalent volt-amperes," but actually greater than the actual sum of the VA in the three lines of the supply (see Appendix). In most commercial supplies this discrepancy between the readings of indicators of the several types would be so small as to be practically negligible.

Numerical Illustrations.

We conclude this chapter with some fully-worked numerical examples of calculations of power factor from data obtained by the methods that have been discussed.

(1) The current in a 3-phase balanced 400/230 volt supply is 25 amperes. The disc of a meter

measuring the supply is found to execute 20 revolutions in 72 seconds. The revolutions-per-unit (R.P.U.) gearing constant of the meter is 80. Find the power factor

$$\text{kVA} = \frac{3 \times \text{phase voltage} \times \text{amperes}}{1000}$$

$$= \frac{3 \times 230 \times 25}{1000} = 17{\cdot}23.$$

Using the formula on page 41:

$$\text{kW} = \frac{20 \times 3600}{80 \times 72} = 12{\cdot}5.$$

$$\text{Power factor} = \frac{\text{kW}}{\text{kVA}} = \frac{12{\cdot}5}{17{\cdot}23} = 0{\cdot}695.$$

(2) A balanced 3-phase load is measured by the 2-wattmeter method. The reading of one instrument is 1200 watts forward, that of the other is 200 watts reverse. Find the power factor.

There are three ways of solving this problem:

(*a*) By the formula

$$\tan \phi = \sqrt{3} \times \frac{W_1 - W_2}{W_1 + W_2}$$

where W_1 and W_2 are respectively the greater and lesser wattmeter readings and ϕ is the angle of which $\cos \phi$ is the power factor. Remembering that a reverse reading is negative we have, applying the formula:

$$\tan \phi = \sqrt{3} \times \frac{1200 - (-200)}{1200 + (-200)} = \sqrt{3} \times \frac{1400}{1000} = 2{\cdot}42.$$

From trigonometrical tables we find that the angle whose tangent is 2·42 has a cosine of 0·38. This is the required power factor value.

(*b*) By the formula

$$\text{Power factor} = \frac{n+1}{2\sqrt{(n^2-n+1)}}$$

where n is the ratio of the greater to the lesser reading of the wattmeters.

From the data given $n = \dfrac{1200}{-200} = -6$. Substituting this value in the formula, and remembering that the square of n is always positive, we have

$$\text{Power factor} = \frac{-6+1}{2\sqrt{\{36-(-6)+1\}}} = \frac{-5}{2\sqrt{(36+6+1)}}$$

$$= -\frac{5}{2\sqrt{43}} = 0{\cdot}38$$

we ignore the negative sign of the penultimate fraction, because power factors are always positive, and we can obtain a positive answer by taking a negative value of the square root.

(*c*) By the direct method:

$$\text{kW} = \text{algebraic sum of readings} = \frac{1200-200}{1000} = 1$$

$$\text{kVAr} = \sqrt{3} \times \text{algebraic difference of readings}$$

$$= \sqrt{3} \times \frac{1200+200}{1000} = 2{\cdot}42.$$

$$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAr})^2 = 1 + (2{\cdot}42)^2 = 6{\cdot}89$$

$$\text{kVA} = \sqrt{6{\cdot}89} = 2{\cdot}62$$

$$\text{Power factor} = \frac{\text{kW}}{\text{kVA}} = \frac{1}{2{\cdot}62} = 0{\cdot}38.$$

Methods (*a*) and (*b*) are those found in the text-books. The first method is concise and convenient if a table of trigonometrical ratios is available, but not otherwise, although there is an awkward and not easily remembered method of finding the cos ϕ corresponding to a given value of tan ϕ by means of a 10-inch slide rule. The second method (*b*) is an academic one. It involves memorising a complicated formula, and the possibility of error when n is negative, and it is best avoided. The third method depends upon two basic rules, the one for finding watts and VAr from the instrument readings and the other for finding VA from watts and VAr. The method is thus straightforward and is best for practical purposes.

(3) Two wattmeters measuring a 3-phase balanced load are each used with a 50/5 current transformer. The instrument readings in watts are 1000 and 400 both forward. Find the 3-phase kW, kVA, and power factor.

When a wattmeter is used with a current transformer the instrument carries a fraction of the supply current which depends upon the transformer ratio. A ratio of 50/5 means that the

supply current is 10 times the wattmeter current. The corrected wattmeter indications are therefore 1000×10 watts $= 10$ kW and 400×10 watts $= 4$ kW.

$$\text{3-phase power} = 10 + 4 = 14\text{kW}$$
$$\text{3-phase kVAr} = \sqrt{3} \times (10 - 4) = 6 \times \sqrt{3} = 10{\cdot}4$$
$$\text{3-phase kVA} = \sqrt{(16^2 + 10{\cdot}4^2)} = \sqrt{(256 + 108)} = 19{\cdot}1$$
$$\text{Power factor} = \frac{14}{19{\cdot}1} = 0{\cdot}73.$$

(4) The kWh and kVArh consumptions in a balanced 3-phase supply are measured by the approximate methods described on pp. 48 and 50. The advance of the kWh meter is 32, and that of the kVArh meter is 30, over the same period. Find the average power factor.

$$\text{kWh consumption} = 3 \times 32 = 96$$
$$\text{kVArh consumption} = 30 \times \sqrt{3} = 52.$$

These consumptions are proportional respectively to average kW and kVAr. A conventional average kVA value is therefore

$$\sqrt{(96^2 + 52^2)} = 109{\cdot}5.$$

$$\text{Average power factor} = \frac{96}{109{\cdot}5} = 0{\cdot}87.$$

CHAPTER III

HOW LOW FACTOR ARISES

The Power Factor of an Electric Supply.

In Chapter I we considered the subject of power factor in a rather academic way by the study of electrical circuits of an artificial character. We saw that a lagging power factor arises from inductance and that inductance is a kind of measure of the magnetic field linking with a circuit per ampere of current carried by it. Low lagging power factor in an A.C. circuit means that the supply to this circuit is compounded of kW, representing the rate of power dissipation, and kVAr, which may be considered as producing alternating magnetic fields.

As we shall explain more particularly in Chapter V the power factor of the whole supply to a group of consuming devices is governed by the electrical characteristics of all the devices in use. Those devices that require no kVAr will tend to keep the overall power factor high. If a consuming device draws VAr as well as kW from the supply system, the power factor of its individual supply will be less than unity, and this will make the overall power factor less than unity. We shall

consider in some detail the various consumers' apparatus that draw kVAr from a supply system and so tend to lower the resultant or overall power factor of a consumer's supply.

Transformers.

A static transformer is a machine used to alter the pressure of an A.C. supply. It consists essentially of an iron core encircled by two windings. One, the primary, connected to the source of supply and the other giving the secondary supply. The ratio of primary supply pressure to secondary pressure is approximately equal to the ratio of primary to secondary terms. The secondary pressure is produced by the alternating magnetic flux in the core, which flux is set up by a current drawn from the primary supply. When no supply is taken from the secondary winding, the primary input is this magnetising or exciting current. When the secondary supplies load, this load current tends to destroy the magnetism of the core, and, to correct this, the input to the primary is increased. The additional input to the primary due to secondary loading corresponds in kVA and power factor to the secondary output. Because of the exciting current, and the magnetising kVAr it contains, the power factor of the total primary output must be less than that of the secondary output, provided that the secondary power factor is not leading.

The equality of additional primary input to secondary output only obtains however if the transformer is so constructed that magnetic flux set up by current in one winding passes entirely through the other. If this is not the case, the flux that only passes through the magnetising winding is called a leakage flux. In a transformer with leakage, the additional primary input in kVA must be greater than the secondary kVA output, because of the additional current required to overcome the demagnetising effect of the secondary current. The effect of leakage is exactly that of an inductance connected between the primary of a non-leaky transformer and the supply, and it is therefore to reduce the secondary pressure with constant supply pressure and to make the power factor of the primary input less than that of the secondary output. Each of these separate effects increases as the secondary output increases. Thus the power factor of the input to a transformer is less than that of the secondary output from two causes (*a*) the exciting current required to provide the core magnetic flux, and (*b*) magnetic leakage.

Magnetic leakage is represented quantitively by the reactance voltage of a transformer, which is the voltage drop expressed as a percentage of the supply pressure that would occur in the equivalent series inductance, mentioned in the preceding paragraph, with the secondary of the transformer

delivering its full load current. Suppose that the reactance voltage of a transformer is 5 per cent. If the secondary delivers 100/5 or 20 times full load current the actual reactance voltage, or the pressure drop in the equivalent inductance, will be equal to $20 \times 5 = 100$ per cent. of the supply pressure. Twenty times full load current is therefore the maximum that can be obtained by short-circuiting the secondary terminals. Thus, percentage reactance voltage determines the maximum current that can be obtained from the secondary of a transformer.

Practical Applications of Transformer Leakage.

Transformers for normal duty are designed with small leakage. In order to avoid considerable fall of secondary pressure at full load, a common value for the reactance voltage is 5 per cent. There are however special duties for which a high transformer reactance voltage is desirable. A good example of special duty of this kind is found in the high pressure supply to long neon tubes, which may be several thousands of volts. The pressure required to start the discharge in the tube is much greater than that required to maintain it. The transformer used to obtain the high pressure supply is designed to give sufficient voltage to start the discharge, and also with considerable leakage.

When the transformer is connected to the A.C. supply and before the discharge starts there is no secondary output and the full secondary pressure is obtained. Immediately the discharge is initiated the secondary delivers current, and the effect of the high reactance is to reduce the secondary pressure to the value required to maintain the discharge.

Transformers used for supply to resistance welders, electric arc furnaces, and for A.C. arc welding are designed with considerable leakage and high reactance voltage in order to limit the maximum current when the secondary windings are partially short circuited.

When an intentionally leaky transformer has its secondary winding short circuited, the current from the supply is limited mainly by what is equivalent to an inductance in series with the primary winding. The power factor of the input is therefore very low.

A.C. Motors.

All electric motors consist essentially of two components, one fixed and the other rotating. The motor torque is produced by the interaction of the currents in one member on the magnetic field associated with the other. If an A.C. motor draws the whole of its supply from A.C. mains, the magnetic field will be of an alternating character, and to produce this field, lagging kVAr will be

required unless the motor is provided with some auxiliary feature whereby these kVAr can be, so to speak, generated by the machine itself. A synchronous A.C. motor has two separate supplies, the one A.C. for producing the power, and the other D.C. for producing the magnetism, and this kind of motor can be made to work without drawing any VAr from the supply. The power factor of the supply to all A.C. motors excepting those of the synchronous type is therefore lagging, because of the VAr that are essential for the production of the magnetic field required for the driving torque, unless the special auxiliary features just referred to are incorporated in the machine.

Induction A.C. Motors.

As the induction motor is the most robust of all types of A.C. motors, it is used to a much greater extent than any other type. Indeed, most industrial power installations supplied on the A.C. system consist entirely of induction motors. It will therefore be desirable to study in some detail the subject of the power factor of the supply to these machines, in order that a clear understanding may be obtained of how this power factor depends upon the loading of the motor and upon the speed for which it is designed.

The stator or fixed component of an induction motor normally contains the winding that is

connected to the supply system. This winding may be considered as having two functions. First, it produces the magnetic field required for the operation of the machine. This field primarily alternating is, in a 3-phase machine, made constant in value and rotating in space by the combination of three alternating fields. The kVAr taken from the supply for the production of this magnetic field are quite considerable, and may be of the order, in magnitude, of one third of the full load watt input. The second function of the stator winding is like that of the primary winding of a transformer—it conveys current by electromagnetic induction to an isolated winding on the rotor or moving component of the machine. The interaction of rotor currents and stator field produces the driving torque.

As in a transformer, the component input to the stator ~~rotor~~ that is transferred inductively to the rotor will have a power factor about the same as that of the rotor currents. Now, as the load on an induction motor increases, the rotor reactance increases; because of the drop in speed, or increase of slip. Thus the rotor kVAr, which of course are derived ultimately, through the stator from the supply, increase more than proportionally to the load.

The total kVAr input taken by an induction motor may therefore be considered to be made up of two components, the one constant at all

loads to supply the stator field, the other increasing more rapidly than the load, that is transferred by transformer action to the rotor. Now, were the first component only present in the supply, an increasing kW input would be associated with

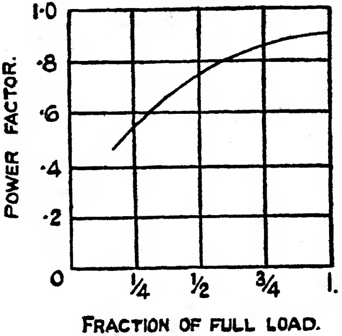

Fig. 19.

a constant kVAr input, and the power factor would always rise as long as the load increased. Because of the second kVAr component however, this rise is checked when a certain load is reached, and, thereafter, further increase of load reduces the power factor. The load giving maximum power factor is usually about or greater than full load.

Figs. 19 and 20 show typical graphs of power factor variation with load of induction motors. Fig. 19 shows that the maximum power factor of the motor occurs well above full load. The machine to which fig. 20 applies has its maximum

power factor at about full load. The load relative to full load at which the maximum power factor occurs depends upon details of the design of the machine.

It is evident from a brief study of these curves that the power factor of the supply to a motor for

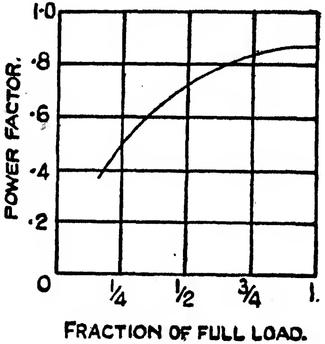

FIG. 20.

a given horse-power output depends very largely upon the size of motor chosen for the duty. If the required output corresponds to the rating of the motor, so that the machine works with full load, the power factor will have about its maximum value. If the motor chosen has its rating much in excess of the duty imposed upon it, the power factor of its supply will be considerably lower.

The full-load power factor of an induction motor of stipulated horse-power rating will depend to a considerable extent upon its rated speed. The normal speed of a 3-phase induction motor is very

nearly equal to the speed of the rotating field. This field speed is known as the synchronous speed and it depends only on the frequency of the supply and upon the arrangement of the stator winding. Now, it will be evident on reflection, that the slower the speed at which a motor delivers a given horse-power output the greater is the torque it must develop, and as torque depends upon current and magnetic flux, the greater is the magnetic flux required. As alternating flux is produced by kVAr from the supply, it follows that a slow-speed motor will require more kVAr per horse-power of output than a high-speed machine and the full-load power factor of the slow-speed machine will therefore be lower than that of a high-speed machine of the same horse-power.

The following table shows how the power factor of a 3-phase 10 h.p. induction motor varies according to the synchronous speed for which it is designed.

Synchronous Speed.	Power Factors.		
	Full Load.	¾ Load.	½ Load.
r.p.m.			
3000	0·90	0·87	0·78
1500	0·89	0·85	0·76
1000	0·87	0·82	0·72
750	0·80	0·73	0·60
600	0·77	0·70	0·58

The foregoing discussion has shown the causes of the wide variation possible of the power factor of the input to an induction motor. It appears from this discussion that in order to obtain the maximum power factor for a motor for a stipulated horse-power duty, the rating of the motor should be chosen so that it corresponds as closely as possible to this duty, and that the speed of the machine should be fixed at the highest convenient value. A synchronous speed lower than 1000 r.p.m. may be advantageous in certain conditions, but the advantage is obtained at the expense of the draw-back of lowered power factor. The full-load power factor of a motor of given speed depends to some extent upon its horse-power rating. Thus the full-load power factor of a 10 h.p. 1500 r.p.m. machine is seen from fig. 21 to be 0·89. The full-load power factor of a 3 h.p. machine of the same speed would be lower, say 0·82.

Single-phase Induction Motors.

The power factor of these machines is inherently lower than that of 3-phase motors of equivalent rating and speed, because the rotor as well as the stator of a single-phase motor is magnetised. Modern single-phase motors are often started by means of a condenser connected in circuit with the auxiliary winding, and part or all of this condenser may be left permanently in circuit. The

effect of the permanent connection of a condenser is to raise the power factor of the supply to the motor in a way that has already been explained briefly in Chapter I, and which will be dealt with more fully in Chapter V. A single-phase induction machine started by the use of a condenser is usually called a capacitor motor.

Synchronous Motors.

We have already seen that as these machines use direct current for the production of the magnetic field in them, they can operate without drawing kVAr from the supply mains. We shall see also in Chapter V that a synchronous motor can be used as a generator of the magnetising kVAr of induction motors, so that the kVAr drawn from a supply system for a power installation containing both types of machines can be reduced.

A.C. Commutator Motors.

An A.C. commutator motor like an induction machine requires an alternating magnetic field; but in some kinds of motors of this type the commutator is used in a peculiar way, difficult to explain, the effect of which is to make the machine produce part or all of the magnetising kVAr required for the working magnetic field. A typical machine of this kind is the B.T.H. A.C. Variable-Speed Commutator Motor. This machine is essentially an induction motor into the secondary

windings of which variable voltages are injected. The magnitude of these voltages determines the slip and hence the speed of the machine. Further, the injected voltages can assist as well as oppose the voltages induced by ordinary induction motor action, so that speeds above as well as below synchronism. The variation of the value of the injected voltages is obtained by adjustment of the separation of pairs of brushes bearing on a commutator, and the fundamental office of the commutator is to convert the frequency of the injected voltages from that of the supply to that of the voltages produced in the secondary by ordinary induction motor action. By rotating the brush system as a whole round the commutator, the phase of the injected voltages can be varied and the machine can be made to produce part or all of the kVAr required for its working magnetic field. This kind of brush adjustment can, however, only be made to suit certain conditions of operation, in other words, it is not possible for the motor at all times to generate the whole of the magnetising kVAr it requires. A fuller explanation of this feature of an A.C. commutator motor will be found in Chapter V.

The curves in fig. 21 give an idea of the power factors of the supply to a motor of this class in various conditions of load and speed. They apply to a machine having a normal "induction motor"

speed of 500 r.p.m., which can be raised to 720 r.p.m. or lowered to 240 r.p.m. by adjustment of the separation of brushes. The maximum outputs at the highest and lowest speeds are respectively 80 and 27 h.p.; these h.p. values, of course, correspond

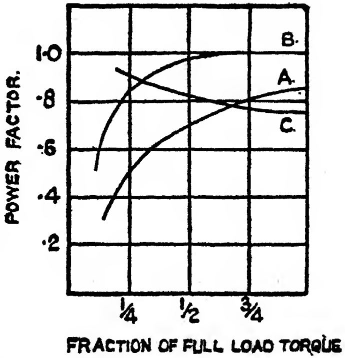

FIG. 21.

to a fixed maximum torque. Curve A applies to the condition when the machine operates at just under 500 r.p.m. as an ordinary induction motor. The special features of the machine are out of use, so that no modification of the natural induction motor power factor is possible. Curve B shows the variation of power factor with load at the highest speed. It is seen that in this condition of working the power factor is approximately unity from half to full load. Curve C applies to the condition of minimum speed.

Summary.

We conclude this chapter by classifying ordinary consuming devices accordingly as they tend to raise or lower the overall power factor of the supply to an installation in which they are included:

(*a*) Approximately unity power factor, tending to raise overall power factor:—

- Wire filament lamps.
- Electric heaters and fires.
- Water heaters.
- Cooking devices.
- Synchronous motors.
- Capacitor motors, possibly.
- A.C. variable-speed commutator motors at certain speeds.

(*b*) Lagging power factor less than unity, tending to lower overall power factor:—

- Vacuum discharge lamps.
- Welding plant.
- Electric furnaces.
- Any apparatus requiring a transformer.
- Ordinary induction motors.
- A.C. commutator motors operating at speeds near the synchronous value.

No ordinary consuming device takes an A.C. supply at a leading power factor, unless specially so designed.

CHAPTER IV

POWER FACTOR TARIFFS

Cost of a D.C. Supply.

So far we have considered the subject of power factor from the physical point of view. We have now to deal with a different aspect of this subject—the financial, and to consider how the cost of an A.C. supply depends upon its power factor. From this consideration we shall pass on to a study of systems of charging for an A.C. supply under which the overall or average rate per kWh depends in some way on the power factor.

Let us first consider what is not very common now, a power station isolated from all other sources of supply and generating and distributing direct current to the area surrounding it. The authority owning the power station and the distributing mains must carry on its business in such a way that the revenue obtained from its consumers is at least equal to the total cost of giving the supply, and the rates of charging for consumers' supplies must be so fixed as to lead to this financial result.

The total cost of generating and distributing a

supply to consumers is of a very complex character. It is however divisible into two distinct components. The first of these components depends upon the energy in kWh actually generated in the power station. This component forms what is known as the running costs and it can be stated as a rate per kWh. If, for instance, this rate is $\frac{1}{3}$d. per unit, this means that the total cost of the supply is increased by $\frac{1}{3}$d. for each extra kWh generated.

The running costs are however only a fraction of the total costs. The provision of generating plant and distributing mains involves costs that have nothing to do with the output in kWh. These costs must be met whether consumers take the supply or not. If the supply system is owned by a municipality, interest must be paid on the money borrowed to provide the generating and distributing plant, and a sum equal to a certain percentage of the borrowed money must be set aside each year to extinguish the loan. The conditions with a company-owned system although apparently different are really quite similar. The company has to provide profit for distribution as dividend to shareholders and to set aside a sum each year against depreciation and obsolescence of plant. In each case what may be called the capital costs represent a more or less definite percentage of the capital expenditure, and as this capital

expenditure is approximately proportional to the capacity of the whole plant in kW of power supply, these capital costs can be represented as a yearly amount per kW of maximum load that the plant can carry continuously. This, however, is not all. There are many other costs that have to be met irrespective of the actual output. Rates, taxes, and insurance are items of this kind. Further a management and maintenance staff must be permanently engaged to direct and supervise the business of the undertaking. Lastly, in order that the steam plant driving the electric generators in the power station may always be ready to meet possible power demands, a certain amount of fuel must be used that is really irrespective of the actual kWh output. The totality of all these costs which have to be met by the supply undertaking irrespective of the actual output is known as the standing costs, and it can be expressed as a yearly amount per kW capacity of plant.

The total cost of giving a supply from a power station to private consumers is therefore made up somewhat as follows:

(*a*) Running costs, proportional to kWh output, and expressed as a rate per kWh:

Fuel (part).

Oil and water.

Repairs and maintenance (part).

Wages of workmen (part).

(*b*) Standing costs, proportional to plant capacity, and expressed as a rate per kW per year:
- Capital and depreciation costs.
- Rent, rates, taxes, and insurance.
- Repairs and maintenance (part).
- Salaries of staff.
- Wages of workmen (part).
- Fuel (part).

Plant Load Factor.

If the standing costs of a supply undertaking are expressed symbolically as £A per kW per year, and the running costs are B pence per unit, then the total yearly costs in pence will be

$$240A \times (kW) + B \times (kWh)$$

where (kWh) stands for the total output in a year. The average cost per kWh in this year will be, in pence,

$$240A \times \frac{(kW)}{(kWh)} + B.$$

We have seen in Chapter I that a kWh output is a measure of the average kW rate of output. Thus if we divide (kWh) by 8760, the hours in a year, we shall obtain the average kW load on the power station. Otherwise (kWh) = 8760 × (Av. kW), so the average cost per kW of output will be

$$\frac{240A}{8760} \times \frac{(kW)}{(Av.\ kW)} + B.$$

The ratio of average kW to actual kW of plant capacity is called the plant load factor. Denoting this quantity by L the overall average cost per kWh of output is

$$\frac{240A}{8760L}+B.$$

The quantity L depends upon the way the capacity of the whole supply system is used by the

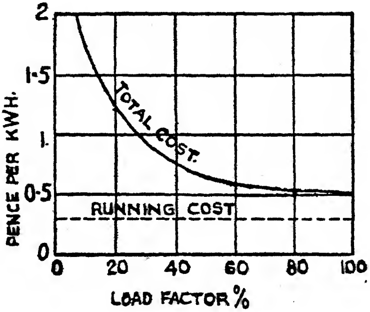

Fig. 22.

private consumers. A load factor of 1 would correspond to the practically impossible condition of the plant being fully loaded throughout the year; but in this condition the output would be a maximum. It is however possible for the whole plant capacity to be used for a limited time. The plant load factor evidently depends upon the greatest demand upon the plant, and also upon the time during which this demand lasts.

The overall cost of a kWh of output depends very largely upon the quantity L because the standing component of the total costs per kWh is inversely proportional to it. The nature of the variation of this overall cost with plant load factor is shown in fig. 22.

Losses.

Only a fraction, of the order of 80 per cent., of the total output of a power station actually reaches the service terminals of consumers. The residue is lost in heating the conductors transmitting and distributing the supply. The total losses depend not only upon the output, but also to some extent upon the plant load factor since they are proportional to the square of current. Thus, if the same output is delivered in half the time, the current will be doubled but the rate of loss will be quadrupled, so that the loss per kWh will be doubled.

The effect of electrical losses is first to make the standing charge per kW of the supply available to consumers greater than the standing charge based on plant capacity, because, owing to pressure drop, only a fraction of plant kW capacity can be used by consumers. The second effect of losses is to make the running cost per kW supplied greater than the corresponding cost per kWh generated, because slightly more than a kWh must be generated for each kWh received by consumers.

Use of the Supply.

We have seen how the overall cost of production per kWh depends upon the way consumers as a whole make use of the supply, and it thus appears that the amount each consumer should pay for his electrical energy ought to depend in some way on the characteristics of his individual use of the service offered by the undertaking.

The most important of these characteristics is, of course, energy consumption in kWh, and as most consumers use the supply to obtain energy, this is the characteristic that interests them. From the point of view of cost of production, the maximum rate at which a consumer uses his supply is also very important, because this rate is evidently of some relation to the demand he makes upon the total plant capacity of the undertaking.

The maximum rate at which a consumer uses the supply, measured in kW, is called his maximum demand. The ratio of his average to his maximum demand over a period is called the consumer's load factor corresponding to this period. Thus for a year period the average load is $\frac{(\text{kWh})}{8760}$, and the load factor is $\frac{(\text{kWh})}{8760 \times (\text{max kW})}$.

It is important to note that if any two of these quantities are known the third can be found. If

kWh consumption is known, and load factor can be estimated with fair certainty, (max kW) is also known approximately.

Two-part Maximum Demand Tariffs.

These tariffs, applicable mainly to power supplies, are based upon what is called the Hopkinson principle of charging for the supply. This principle is that consumers should contribute towards the running costs of the undertaking in proportion to their kWh consumptions, and towards the standing costs in proportion to their maximum kWh demands. A tariff based on this principle therefore includes two price rates, the one per kWh consumed, and the other per kW of maximum demand, per month, per quarter or per year as the case may be. This is known as a two-part tariff. The physical quantities involved in this tariff are measured, the one by a meter, and the other by a kW demand indicator of one of the types briefly described in Chapter III.

The Hopkinson system of charging is intended to make each consumer's bill equal to the actual total cost of producing and delivering his supply. This object is only partially attained. The actual demand that a consumer makes upon the plant capacity of the power station supplying him depends not only upon his measured maximum demand but upon the time that this maximum

occurs. If the maximum demands of a group of consumers—say those using the supply for industrial power purposes—do not occur simultaneously, the resultant demand of the whole group will be less than the sum of the separate measured maximum demands. A consumer whose maximum demand occurs at a time when all other consumers are only making small demands on the supply will not increase the actual total maximum demand on the plant at all. This effect, whereby different times of incidence of individual demands makes the resultant maximum demand less than the sum of the individual maxima, is called diversity, and the ratio of the sum of the maxima of a group of consumers to their resultant maximum demand is called the diversity factor of the group. Unlike an individual or a plant load factor the diversity factor of consumers classified according to their use of the supply cannot, as a rule, be measured. If, however, a value of diversity factor can be estimated, then, averaging its effect over the group of consumers concerned, each measured kW of maximum demand can be considered to be equal to a fraction of a kW of effective demand which is equal to 1 divided by the estimated diversity factor.

The two-part maximum demand tariff is therefore not quite so scientific as it appears to be at first sight, because of this unmeasurable effect of

diversity. It is however a reasonably equitable method of charging for a supply because it does with some measure of approximation require the consumer to contribute towards both components of the total costs of the supply according to a rational scheme.

Other Tariffs.

The flat rate tariff, whereby the whole charge for the supply is based on kWh consumption has the merit of simplicity, and it is justified, on the Hopkinson principle by assuming a load factor and a diversity factor applicable to the uses of the supply to which the kWh rate applies. As we have explained previously, if a consumer's load factor is assumed to be known, then his kWh consumption is a measure of his maximum demand. Thus by assuming load factor and diversity factor, the demand rate can be converted into a kWh rate which is merged into the running charge to give an overall flat rate per kWh.

Two-part domestic tariffs have nothing to do with the subject of power factor, but we may conclude our discussion of the Hopkinson theory of charging by saying that a fixed quarterly charge based upon size of house, rateable value, or kW rating of apparatus installed is sometimes supposed to be justifiable by this theory on the ground that the quantity used for the fixed charge assessment

is proportional to the maximum demand the consumer will make. The assumption here is very questionable and hardly reasonable, and domestic tariffs cannot be considered to be justifiable by any scientific system of charging. Actually they are framed with a view to encouraging the use of electricity for domestic purposes other than lighting, and it is possible that their practical effect is that many large domestic consumers are unremunerative and are supplied at a loss, which of course has to be borne by those consumers who pay for their supply under tariffs of a more scientific character.

Cost of an A.C. Supply.

The whole of the preceding discussion has been concerned with a D.C. supply from an isolated power station, but it will apply as it stands to an A.C. supply, always of unity power factor, of which 1 ampere of current at the standard voltage always gives the same amount of power. If however the power factor of an A.C. supply falls below unity, the overall cost of supply per kWh is affected considerably. To clarify our inquiry into this matter let us assume that the power factor of the A.C. output of a power station has the constant value of 0·5. This means that the current and kVA per kW supplied will be double what it would be if the power factor were unity.

Now the maximum electrical load that can be carried by any A.C. machine, cable, or transforming device is based upon the maximum current that the machine, cable, or transformer can carry continuously without being dangerously overheated. Thus maximum electrical load, or rating, cannot therefore be specified in kW at a definite voltage as it can with a D.C. supply in which the current per kW is fixed; it must be specified in kVA at the standard voltage. The capital cost of all electrical machinery and cables for an A.C. supply is therefore proportional to its kVA rating and not to the kW of load it will deal with. If therefore an A.C. supply is given at 0·5 power factor, the electrical plant rating in kVA will be double that required for a unity power factor for the same power in kW, and the cost of this plant will be approximately doubled.

This increase of requisite rating and consequently of cost per kW applies only to electrical plant, and not to the prime movers or to the steam-generating plant in the power station. The steam per hour required by a turbine for a given kW output of the alternator it drives is nearly independent of the power factor of this output; actually the steam consumption will increase slightly per kW as the power factor falls because of the reduced efficiency of the alternator, but this increase will be quite small.

It appears therefore that, for a given kW output of A.C. at 0·5 power factor, a large portion of the capital costs of an undertaking will be doubled because of the increased kVA of capacity required in all electrical components of the system. The capital costs of steam-generating and prime-moving machinery will, however, be unaffected by power factor. This applies also to most of the other items in the list of components of total standing costs given on page 80. It may be said, as a general principle, that a considerable fraction of the standing costs of an undertaking supplying A.C. is inversely proportional to the power factor. If, as is usual, the power factor is not constant, then the variable fraction of the standing costs may be considered to be inversely proportional to the power factor of the power station output at the time of its maximum output.

This increase of the standing costs is the most important effect of low power factor on the overall cost of A.C. supply per kWh. A secondary effect is that of increased losses. A power factor of 0·5, meaning as it does that the current per kW is doubled, will entail four times the loss in distribution and in electrical machinery handling the supply. Further, owing to the increase in pressure loss set up by the increased current, the kVA output per kVA delivered at consumers' services will be increased. These effects of the increased losses

due to low power factor will slightly increase the standing costs per kW and the running cost per kWh, delivered to consumers' terminals.

The Cost of Bulk Supplies.

Supply authorities now purchase their energy in bulk from the Central Electricity Board, on the 3-phase A.C. system. This energy is paid for under a two-part tariff based upon kWh supplied and upon measured maximum demand in kW corrected for power factor. The power factor at the time of the maximum demand is determined from a chart record of kVAr demand furnished by a special meter provided for the purpose. The rate payable by the undertaking per kW of maximum demand on the Grid therefore rises as the power factor of the maximum demand falls.

Power Factor Tariffs.

The additional cost of a supply caused by low power factor must, of course, be recovered by a supply authority from its consumers. It would of course be possible to adjust normal tariff rates so that this increased cost is borne by all consumers irrespective of whether or not their individual supplies helped to cause the low power factor. Such a policy would, however, be in conflict with the underlying idea of the Hopkinson theory according to which the charge for the supply ought

as far as possible to correspond to the total cost of producing and delivering it. According to this idea a consumer whose use of the supply is such that his power factor is low ought to pay more per kWh than another consumer, using the supply similarly in all respects excepting that his power factor is unity.

Some supply engineers, in discussing this matter of recovering from consumers the additional costs incident to low power factor, have spoken of penalising the low power factor consumers. This idea of penalising or punishing a consumer for taking a supply at low power factor is fundamentally wrong. Reactive kVAh are offered to the consumer by every undertaking supplying A.C., and these kVAr are used, as we have seen, for the essential purpose of supplying the alternating magnetic fields that are necessary for the operation of electrical machinery. The power consumer does not directly require the kVArh in his supply which cause low power factor—he requires kWh for producing mechanical energy, and to produce this mechanical energy, the reactive kVArh supply is indirectly necessary. To talk of penalising the consumer for taking the reactive supply necessary for the production of his mechanical energy by ordinary electric motors is therefore as wrong as to talk of penalising him for taking kWh. All that can equitably be done is to require the consumer

to pay approximately the increase in the cost of producing and delivering his supply which is caused by his low power factor.

We may cite, as an example of a "penal" method of charging for low power factor supplies that has been proposed, the flat rate per kVAh. According to this method a consumer taking his supply at 0·5 power factor would pay double the kWh rate applicable to another consumer whose power factor was unity. In effect this system of charging would not only double the running cost recovered from the consumer, which running cost is very little affected by power factor, but would double his contribution to the standing costs which are only partly affected by power factor. A system of charging like this is plainly inequitable; so far it has not been used by supply engineers, not so much because of its unfairness, but because a cheap and simple A.C. kVAh meter cannot be made.

A system of charging, under which a consumer is required to contribute approximately the extra cost of his supply caused by the low power factor of it, is often called a "power factor tariff." The object of a tariff of this kind is sometimes conceived to be an inducement for the consumer to increase his power factor, or, in other words, to reduce his kVAr demand per kW, by one of the methods that will be described in the next chapter. A power

factor tariff generally offers this inducement, but not always, and the fundamental object of the tariff is rightly conceived to be merely an equitable adjustment of the overall kWh rate to meet the increased cost of supply. The inducement for increase or improvement of power factor is secondary. As a matter of fact it is quite as possible for an undertaking to supply idle kVAr profitably as it is to supply kWh, but, as we shall see, it is usually cheaper for the consumer to produce the kVAr he requires by his own plant than to buy it, under a power factor tariff, from the supply authority.

The Two-part Maximum kVA Demand Tariff.

This method of charging is an attempt to equate the increased overall rate per kWh to the increased cost of supply due to low power factor. The maximum demand is assessed in kVA and part of the charge is quoted as a yearly, quarterly, or monthly rate per kVA of measured maximum demand. The second component of the charge depends upon the kWh consumption. Thus, a tariff of this kind might be: £1 per quarter per kVA of maximum demand, and 0·5d. per kWh. A consumer whose kVA demand was 125, and whose consumption was 72,000 units in a quarter would pay £$(125+36000/240)=$£275, and the overall rate per kWh would be $(125\times 240+36000)/72000=0{\cdot}92$d.

The 125 kVA demand depends, of course, not only on the actual power demand on kW, but also on the power factor at the time the maximum occurred. If the actual maximum power demand was 100 kW, the power factor was 0·8. If for the same power demand the power factor was only 0·6, the kVA demand would have been 100/0·6 = 166, and the

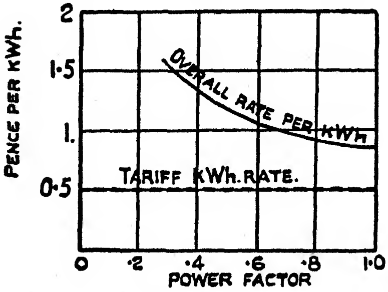

FIG. 23.

power bill would be increased to £166 plus £150 for the kWh consumption making a total of £316—for the same kW of power and the same kWh consumption.

The variation of the overall charge per kWh under this tariff, with power factor changes is shown graphically in fig. 23. The power factor values are those corresponding to the period over which the maximum kVA is recorded. The assumed load factor is 30 per cent.

We have already explained briefly in Chapter III

the underlying principles of the measurement of maximum kVA demand. At the end of each period, year, quarter, or month, upon which the demand charge depends, the indicator is read, and its pointer is then returned to zero so that it is ready to record the maximum demand of the ensuing period. It should be noted that the reading of a demand indicator, unlike that of a meter, is destroyed and cancelled after it is taken and recorded. For this reason it is very important that a representative of the consumer should be present at the time the reading is taken, and he should agree with the reading and the supply authority's record of it before the pointer of the indicator is reset. Power consumers should have a proper understanding on this point with the authorities who supply them.

The two-part maximum kVA demand tariff has the effect of increasing the consumer's contribution to the standing costs of the undertaking in inverse proportion to the power factor of his kW demand. We have seen, however, that the power factor of the maximum output of a power station affects only a portion of the standing costs of supply per kW. The increase of the consumer's contribution to standing costs is therefore, on the grounds of equity, too great. On the other hand, the kWh rate under the two-part tariff is independent of power factor, whereas the running costs of an A.C.

power station increase as the power factor falls. This circumstance is an off-set to the undue increase of the demand charge, so that the tariff as a whole gives a fairly equitable increase of the overall kWh rate per low power factors.

As most supply undertakings purchase bulk supplies from the Grid under a two-part tariff, the kVA demand system of charging consumers corresponds closely to the system under which the undertaking buys its supply, and it is therefore equitable both to the consumer and to the undertaking.

Composite Tariffs.

We now consider a system of charging that is based, not on the Hopkinson maximum demand system, but on a modification of the flat rate. This system depends fundamentally on the idea that low power factor means that the consumer takes from the supply more kVAh than kWh. Thus, if the average power factor of a supply is 0·8, then, for 100 kWh of energy, 125 kVAh will be required, whereas with a unity power factor supply 100 kWh require only 100 kVA. The extra kVA, 25 in the case considered, are generated at the power station without any but very small extra demand on the steam-raising plant. They require practically no fuel, but they require plant and cable capacity to take them to the consumers'

terminals. The cost of an "extra kVAh" will therefore be less than the cost of a kWh. Suppose the flat rate of supply per kWh is A pence, and that of an "extra kVA" B pence, then, according to the principle stated, the charge for the supply will be in pence,

$$A \times (\mathrm{kWh}) + B \times \{(\mathrm{kVA}) - (\mathrm{kWh})\}.$$

As B is less than A, this statement is equivalent to

$$C \times (\mathrm{kWh}) + B \times (\mathrm{kVA})$$

where $C = A - B$.

According to this system of charging, therefore, the consumer pays a flat rate of C pence for his energy in kWh, and a flat rate of B pence for all the kVAh in his supply. B is generally assumed to be $\frac{1}{3}$ of C, so that the tariff charge can be represented by:

$$C \times \{(\mathrm{kWh}) + \tfrac{1}{3}(\mathrm{kVAh})\}.$$

Taking the basic C rate as $\frac{3}{4}$d. the variation of overall charge per kWh with power factor changes is shown graphically in fig. 24. It will be seen that, in comparison with the maximum demand system illustrated in fig. 23, this favours the consumer.

This method of charging for A.C. supplies was proposed many years ago by Arno, an Italian engineer. It has the great merit of being equitable both to the consumer as well as to the supply authority but it suffers from the drawback that

one of the physical quantities appearing in the tariff rates, kVAh, requires a costly instrument for its measurement.

There is, however, another system of charging, requiring the simpler measurement of reactive

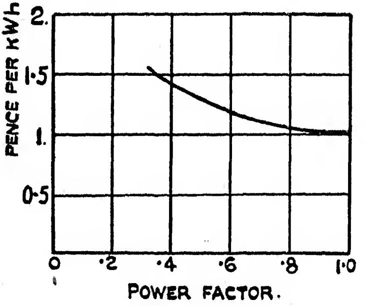

FIG. 24.

consumption or kVArh, which very closely approximates to the Arno system.

We shall show in the Appendix that if we assume or estimate a value for the average power factor, the fictitious Arno consumption is very approximately equal to (kWh) × a constant, plus (kVArh) multiplied by another constant, for considerable power factor variations on each side of the average. Thus if the average power factor be assumed to be 0·71, the approximate equivalent of the Arno consumption will be

$$1{\cdot}235 \times (\text{kWh}) + 0{\cdot}235 \times (\text{kVArh}).$$

The following table gives the exact Arno consumptions and the approximate equivalents based on kWh and kVArh corresponding to this formula for various actual power factors.

Power Factor.	Arno Consumption per kWh.	Approximate Equivalent based on kWh and kVArh.
1·0	1·33	1·235
0·8	1·41	1·41
0·7	1·47	1·47
0·6	1·55	1·55
0·4	1·82	1·785

This table shows clearly that over a very wide range of consumers' power factors a composite consumption based upon kWh and kVArh is very nearly equal to the fictitious consumption based upon the Arno formula. The deviations of the approximate from the true consumption favour the consumer.

We may remark in passing that the fictitious Arno consumption can be registered approximately by a single meter suitably compensated. In this country all consumptions from a public supply undertaking must be registered in kWh whatever be the system of charging, so that this approximate method is not of great practical use, and it is more convenient to measure kWh and kVArh, as these

measurements can be carried out by standard types of meters. (See Appendix.)

In the practical application of the modified approximation Arno system, the charge for the supply is not based upon a composite or fictitious consumption, but kWh are charged for at one rate, and kVArh at another and a smaller rate. Thus, in one actual power factor tariff of this kind kVArh were charged at one-eighth the kWh rate.

This composite kWh and kVArh tariff is not used to any great extent in this country. It has the advantages that it is based upon a rational method of increasing the overall kWh rate for low power factor, that the measurements involved are made with standard meters, and that the idea of paying at a small rate for magnetising kVArh taken from the supply is intelligible to consumers.

Sliding-scale Tariffs.

Under tariffs of this kind the rate per kWh used varies according to the average power factor of the supply. The standard rate is generally based upon an average value of the power factor—such as 0·8. The kWh rate is reduced for power factors above, and increased for power factors below this average value. Under a representative tariff of this kind adjustments to the kWh rate were made on the following scheme.

0·8 power factor: standard rate.

Power factor between 0·8 and 1·0: standard rate reduced by $\frac{1}{2}$ per cent. for every 0·01 the measured power factor is greater than 0·8.

Power factor between 0·8 and 0·6: standard rate increased by $\frac{1}{2}$ per cent. for every 0·01 the power factor is below 0·8.

Power factors below 0·6: standard rate increased a further 1 per cent. for every 0·01 the power factor is below 0·6.

Thus, if a consumer's average power factor were found to be 0·9, then with a standard rate of 1d. his consumption would be charged at 1d. less $10 \times \frac{1}{2}$ per cent., or 0·95d. If the power factor is 0·55, the rate will be increased by $20 \times \frac{1}{2} = 10$ per cent. for the reduction from 0·8 to 0·6, and $5 \times 1 = 5$ per cent. for the further reduction from 0·6 to 0·55, making a total increase of 15 per cent. and a kWh rate of 1·15.

It may be of interest to compare the kWh rates under this tariff with those under the composite tariff with kVArh charged at one-eighth the kWh rate. The comparative figures for equal rates of 1d. at 0·8 power factor are given in the table on page 102.

It is important to note that the power factor used for the adjustment of the kWh rate under a sliding scale tariff is an average value taken over the whole period between two meter readings. This average value will in practice generally be

lower than the power factor at the time of maximum demand that fixes the kVA, and hence the rate per kWh payable under a maximum demand power-factor tariff, excepting in conditions when the consumer installs special apparatus to raise the power factor of his supply.

Average Power Factor.	Sliding-scale Tariff.	Composite kWh and kVArh Tariff.
1·0	0·9	0·92
0·8	1·0	1·0
0·6	1·1	1·07
0·4	1·3	1·11

The average power factor upon which the kWh rate is based in the sliding scale tariff may be calculated from the registrations of separate kWh and kVArh meters. With 3-phase supplies two single-phase meters can be used to measure the kWh consumption and the kVArh, and the average power factor can be obtained from the difference of the registrations of the meters as was explained in Chapter II. We have there stated that this method of measuring kVAr and 3-phase power factor is only accurate for balanced loads. It follows therefore that the application of this method for fixing the kWh rate under a sliding scale tariff is of questionable validity. If however

the use of the supply is for motors and lighting only, the possible unbalance is known to be small. Further, throughout the period between two meter readings the nature of the possible unbalance is likely so to vary that inherent errors in the measurement of kVArh and of power factor very nearly cancel out. The use of separate meters, the one measuring kWh and the other kVArh, is however the best and most accurate method of measuring average power factor.

The principal drawback of the sliding-scale power-factor tariff is the necessity for the calculation of power factor from registered kWh and kVArh consumptions. This calculation is of a technical character and it cannot be grasped or understood by ordinary consumers. On general grounds it is undesirable that the conversion of meter registrations to money values should involve any but simple calculations which the consumer can check if he desires to do so. The evaluation of power factor can, in theory, be simplified by using a graph, but a graph is not a good method of obtaining a quantity upon which a tariff rate and hence a money payment depends.

The sliding-scale tariff, involving power factor, could advantageously be replaced by a similar tariff involving the ratio of kVArh to kWh consumptions. The grading of the kWh rate could be so arranged that the effect of the tariff would

be very approximately the same as that involving power factor. A sliding-scale power-factor tariff of this kind might quote a standard kWh rate for a power factor of 0·8 when the kVArh would be 75 per cent. of the kWh. The kWh rate might be graded as follows:

A 1 per cent. reduction for each $7\frac{1}{2}$ per cent. the kVArh fell below 75 per cent. of the kWh.

A 1 per cent. increase for each 5 per cent. the kVArh rose above 75 per cent. of the kWh.

This tariff, which involves only two adjustment rates is very closely equivalent to the power factor tariff previously quoted, and it has the further advantage that, if kVArh are directly metered, the adjustments to the standard kWh rate, and hence, the total bill, can readily be checked by the consumer.

Summary.

The primary object of power-factor tariffs is to increase the charge for an A.C. supply by an amount bearing some fairly equitable relation to the increased cost of supply due to low power factor. The most rational of these tariffs are the kVA maximum demand tariff that increases the consumer's contribution to the standing costs of the undertaking as his power factor falls, and the composite kWh and kVArh tariff, that increases the overall rate per kWh by an amount correspond-

ing approximately to the "extra kVAh" per kWh taken from the supply by the consumer. These tariffs involve fixed and definite rates. The third kind of tariff involves a sliding scale for the variation of a standard kWh rate, and although this sliding scale is designed to give equitable increases of the actual rate for low power factors, its rational basis is not easily perceived, it often requires complicated calculations for the determination of the power bill, and, from the point of view of the consumer, it has an arbitrary character.

From the point of view of the power consumer, the effect of a power-factor tariff is to increase his power bill if his power factor falls below its maximum value of unity, and this increase is a financial inducement for him to spend money with the object of increasing the power factor, and so getting his supply at a cheaper rate. In the following chapter we shall discuss in detail the various means whereby the power factor of an A.C. supply may be increased.

CHAPTER V

POWER FACTOR CORRECTION

Overall Power Factor.

We have already seen in Chapter I that when an alternating current is out of phase with the pressure producing it, the current is physically equivalent to two currents flowing simultaneously, the one, an active component, in phase with the pressure which conveys all the power, and the other a reactive component, $\frac{1}{4}$ cycle out of phase with the pressure which conveys no power and which is therefore called wattless. If the positive maxima of the actual current occur after those of the pressure, this current is lagging, and the reactive component lags $\frac{1}{4}$ cycle or 90 degrees of angle. If the current is in time advance of the pressure, the reactive component is in advance or leads by 90 degrees. This leads to the idea that an A.C. supply is compounded of kW of power and kVAr of idle reactive apparent power which causes lagging or leading power factor.

Leading kVAr are 180 degrees or $\frac{1}{2}$ cycle out of phase with lagging kVAr. A $\frac{1}{2}$ cycle phase difference is the same as opposition of direction.

It follows therefore that if two circuits are supplied in parallel from an A.C. supply, and the one takes lagging and the other leading kVAr, the kVAr in the resultant supply will be equal to the difference of the component kVAr values, and will lag if the component lagging kVAr be the greater, and vice versa.

Lagging kVAr are always required to produce the alternating magnetic fields essential for the operation of transformers and most A.C. motors, and in the industrial applications of an A.C. supply, these kVAr are the kind normally taken from the supply. Any machine or apparatus that is designed to take leading kVAr from an A.C. supply can therefore be considered to generate or give out lagging or magnetising kVAr.

All consuming devices, lamps, motors, heaters and so forth, supplied from an A.C. system form parallel circuits. The total power taken from the supply is the sum of the power values of the individual consumptions. The total VAr taken from the supply is the difference of the sums of the lagging and the leading individual kVAr consumptions. The overall power factor of a supply to a number of consuming devices in parallel depends upon the total kW and the resultant kVAr consumptions.

This is illustrated graphically in fig. 25, which applies to three circuits each taking the supply at a different power factor. The kW and the lagging

kVAr in circuit 1 are represented graphically by the lines OA_1 and A_1B_1 respectively, drawn at right angles. As has been shown on page 18, the line OB_1 represents to scale the kVA in circuit 1, and the power factor of the supply to this circuit is equal to the ratio OA_1/OB_1. If circuit 2 is now connected in parallel with circuit 1, the kW from the

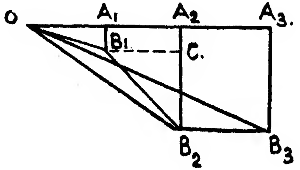

FIG. 25.

supply will be increased to an amount represented by OA_2 if the line B_1C represents the kW used in circuit 2, and if CB_2 represents the lagging kVAr used in this circuit the lagging kVAr taken from the supply will be increased to a value represented by A_2B_2. The power factor of circuit 2 is equal to the ratio B_1C/B_1B_2; the total kVA in the supply is OB_2 and the power factor of this total supply is equal to the ratio OA_2/OB. It is seen from the diagram that the power factor of circuit 2 is less than that of circuit 1, and also that connecting circuit 2 in parallel with circuit 1 has consequently reduced the power factor of the total supply, for the ratio OA_2/OB_2 is plainly less than the ratio OA_1/OB_1.

Suppose the third circuit takes a supply at unity power factor. If the kW in this supply are represented by the line B_2B_3, the total kW in the supply will become OA_3; the kVAr consumption will remain unchanged, and have the value A_3B_3 equal to A_2B_2. The kVA in the total supply now has the value OB_3 and the power factor takes the

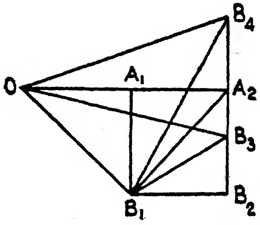

FIG. 26.

value OA_3/OB_3. The power factor of the supply has therefore been increased by adding to circuits 1 and 2 a third circuit of higher power factor.

Fig. 26 illustrates graphically the effect of connecting a circuit of leading power factor to one taking lagging kVAr. Here OA_1, A_1B_1 and OB_1 represent respectively the kW, lagging kVAr, and kVA in the first circuit. When the leading power-factor circuit is connected, the kW from the supply increases from OA_1 to OA_2 where $A_1A_2 = B_1B_2$ represents the kW in circuit 2. B_2B_3 represents the leading kVAr in circuit 2, and these kVAr are in the opposite direction to those represented by

A_1B_1 in circuit 1. Otherwise they may be considered as lagging kVAr generated or sent out by circuit 2 instead of being absorbed or taken in, as with circuit 1. The total kVA in the supply now become OB_3 and the power factor increases from OA_1/OB_1 to OA_2/OB_3. If the leading kVAr in circuit 2 are equal to the lagging kVAr in circuit 1, the net kVAr from the supply is zero, the kVA equals the kW, OA_2 and the power factor of the total supply is unity. If the leading kVAr take a value like B_2B_4, greater than A_1B_1, the net kVAr from the supply A_2B_4 are leading, and the power factor OA_2/OB_4 has a leading value. The combination of the two circuits takes power from the supply, but can be considered as returning lagging kVAr to it.

Avoiding Low Power Factor.

We have already stated that one of the chief causes of the low power factor of industrial loads is the lagging magnetising kVAr required by induction motors. The 3-phase induction motor is so simple and robust a machine for approximately constant-speed duty that it will always be used when the supply is A.C. excepting for special reasons. Once an installation of induction motors is laid down the power factor of the supply is more or less fixed relatively to the horse-power output, and this power factor can only be increased by the

provision of additional plant. When an installation of this kind is being laid down, or extended materially, it is possible, by the observance of the following simple rules, to prevent very low power factor, although, of course, this power factor must necessarily be below unity.

The full load rating of the motor should be as nearly equal as possible to the required mechanical output. This condition, because motor ratings are standardised, can only be met approximately, but it is always well to bear in mind that induction motors can sustain temporary overloads without harm, and that expected peak demands can be handled by the overload capacity of the motor selected. The dependence of the power factor of an induction motor on the load at which it works has been fully explained in Chapter III.

The speed chosen for an induction motor should be as high as possible. Slow-speed machines not only cost more, but have a lower power factor than high-speed motors of equivalent rating (see page 71).

The motors used should be liberally designed. If the amount of iron in a motor is restricted in order to reduce the cost, the kVAr required to magnetise the iron will be increased, and the saving in the cost of the machine will often be more than off-set by the increased charges for the supply because of the lower power factor so caused.

Motors with ball bearings operate at higher power factors than similar machines with sleeve bearings. This is because the magnetising kVAr for a machine of given frame size depends very largely upon the air gap between the rotor and the stator, and a smaller air gap can safely be used with ball bearings.

Single-phase motors of small ratings are often used with a 3-phase supply. Whenever possible these may well be of the capacitor type which give good starting characteristics, and which can be designed to operate at a power factor in the neighbourhood of unity.

A 3-phase motor working at a very low output under 40 per cent. of its rating will have its power factor raised considerably by operating it with the stator windings connected in star instead of in delta as normally. This will reduce the magnetising kVAr input more than proportionally to the reduction of pressure on the stator windings, and so increase the power factor at the low operating load of the machine. The change in the method of connecting the stator windings will reduce the maximum load the machine will carry, so that, if the duty on the motor subsequently increases to over 40 per cent. of the nominal rating, the normal delta connection will have to be restored.

If the motors of a power installation carry practically constant load, and the installation is designed

in accordance with these rules, the power factor of the supply can be expected to be what is considered to be a reasonably high value, *i.e.* in the neighbourhood of 0·8. If, however, fluctuating and intermittent loads have to be handled, the average power factor of an induction motor installation must necessarily be considerably lower than this value. When this is the case, or when a motor installation has been badly designed, a low power factor can be increased only by the provision of special motors or other plant.

Power-Factor Correction.

Power factor correction is the term usually used to describe the increase of the lagging power factor of a supply by connecting to the supply apparatus or machinery one function of which is to generate or give out part or all of the lagging kVAr which cause the lagging power factor. Power-factor correction may be obtained by special motors which, by operating at a leading power factor, generate lagging kVAr as well as produce mechanical power, or by plant the sole office of which is to generate the lagging kVAr, and which takes from the supply no power but that lost in this process.

Synchronous Motors.

Brief reference to this type of motor has already been made in Chapter III. We shall now explain

how it can be made to generate lagging kVAr, whilst operating to produce mechanical power.

A synchronous motor is essentially a reversed alternator. The magnetic field is produced by direct current generally supplied by an auxiliary machine called an exciter. The speed of the motor, like that of an induction machine, depends upon the arrangement of the windings connected to the supply, and it is exactly proportioned to the supply frequency. In other words the motor always runs exactly in step with the alternator providing the supply that drives it. If the average speed differs ever so slightly from this synchronous value, the synchronous motor torque becomes zero.

The action of a synchronous motor may be explained in the following way. The 3-phase supply currents in the motor windings, usually stationary, set up a uniform magnetic field rotating in space at synchronous speed. As long as the magnetic field produced in the rotor by direct current moves at the same rate as the stator field, the two fields will remain in practically the same position relative the one to the other, and the stator field set up by the supply currents will twist the rotor at synchronous speed. If the rotor speed does not correspond to synchronous speed, the two fields will constantly change in their relative position, and the average torque in a revolution will become zero. If the speed of a synchronous

motor falls below synchronism, a small driving torque will be set up by induction motor action, and the machine will not stop or stall unless it is heavily loaded.

A synchronous motor, being constructed exactly similarly to an A.C. generator will, when it is running, produce a back alternating voltage like a D.C. motor, and as the speed is fixed, this back voltage will depend upon the direct current producing the magnetic field. Conceive that this direct current has a low value so that the back voltage of the motor is less than the supply voltage. Current enters the motor windings and the supply to the motor contains a kW or power component and a reactive or lagging kVAr component, because the motor windings are inductive. Imagine now that the direct current, and hence the magnetic field, is gradually increased. The back voltage will rise. The kW power input cannot alter, for, as the speed of the motor cannot vary, it must produce the same mechanical output. The rise of the back voltage must therefore diminish the lagging kVAr input, and raise the power factor of the supply, and as the magnetic field is increased, a point will be reached when the kVAr input becomes zero. When the back voltage of the motor is greater than the supply pressure some kind of current must be returned to the supply. The kW power input cannot vary, so that the output of the motor due

to the high back voltage must be lagging kVAr. In this condition the synchronous motor is said to be "over-excited." An over-excited synchronous motor therefore operates at a leading power factor, and generates reactive or lagging kVAr while absorbing power from the supply. In this way the motor can be used to produce the lagging kVAr required by induction motors and so to correct the low power factor of an installation of A.C. motors.

A synchronous motor may be used for the sole purpose of generating lagging kVAr. The machine, which as a motor runs unloaded, is then called a synchronous condenser.

A rotary converter, driven by A.C., has some of the properties of a synchronous motor, and it will operate at a leading power factor, or generate lagging kVAr, if it is over-excited. In this condition, however, the machine runs inefficiently, and a rotor converter can only be used economically to correct the low power factor of an associated supply by operating it at unity or slightly leading power factors.

The power factor of the supply to a synchronous motor is adjustable by varying the direct current in the field windings, either by a series of resistances or by varying the exciter voltage. The lagging kVAr output depends upon the value of this D.C. and is nearly independent of the load. The leading

power factor, with fixed excitation, will therefore fall as the load on the machine decreases. The cost of a synchronous motor will depend not only upon its normal h.p. rating but also upon the lagging kVAr it is required to produce, because the rotor windings and the exciter output will depend upon this kVAr value.

Modern synchronous motors are of three types. These types differ according to their starting characteristics.

The salient-pole synchronous motor is constructed like an alternator with projecting or "salient" pole pieces, carrying the direct current winding, in which the magnetic flux is located. The magnetic system has attached to it a squirrel-cage winding like an induction motor, and this winding enables the motor to accelerate from rest against light loads to nearly synchronous speed. When the direct current excitation is switched on the speed increases to the synchronous value and the motor "pulls into step." The starting torque of salient-pole motors is poor.

The synchronous-induction motor is constructed like an induction motor with a wound rotor connected to slip rings. It is started as an induction motor with resistance in the rotor circuit, and when the maximum induction motor speed is attained, the rotor is switched over to a direct-current exciter, when the speed rises to the syn-

chronous value. Synchronous-induction motors will start against considerable loads, but they suffer from the disadvantage that the direct-current exciting voltage must be low, and that low voltage exciters are liable to give trouble in operation.

A third modern type of synchronous motor is of the salient-pole construction but it is provided with an additional 3-phase winding on the rotor which is connected to a separate set of three slip rings and used to start the machine from rest as a wound-rotor induction motor. This motor combines the advantages of the ordinary salient-pole and the synchronous-induction types, and as the direct current flows in a winding specially designed for it, the exciter voltage is normal.

Although salient-pole synchronous motors are used principally to produce lagging kVAr for power-factor correction, they offer the important advantages of absolutely constant speed with constant supply frequency, and of high efficiency at all speeds. The cost of a synchronous motor is, of course, greater than that of an induction motor of the same h.p. rating. Synchronous motors of the modified induction type are not so efficient as salient-pole machines.

Phase Advancers.

We have seen, in the preceding section, that an induction motor can be made to generate its own

lagging kVAr, by feeding direct current into the rotor. The machine runs at synchronous speed and with zero slip. The slip frequency is therefore zero, the same as that of the current fed into the rotor.

Any induction motor can be made to produce its own magnetising kVAr by connecting in the rotor circuit a source of supply, the frequency of which is exactly equal to that of the rotor currents, and which has a correct phase difference with that of the rotor voltage. This idea can be grasped by referring to fig. 11 on page 23. The voltage drop on the inductance X is $\frac{1}{4}$ cycle out of phase with the current, and this voltage may be conceived to be the cause of this current in the resistance R lagging the supply pressure instead of being in step with it as would be the case if X were not in the circuit. The voltage on X could be replaced by a voltage similar in magnitude and phase, derived from another source of A.C. supply, and this second or auxiliary voltage would make the current in R lag in phase on the main voltage. Reversing the auxiliary voltage would make the current lead in phase. An auxiliary voltage like this, which modifies the phase of a current, is often called an "injected" voltage. A phase advancer is an auxiliary machine to an induction motor, the function of which is to produce a voltage of the same frequency as that of the rotor currents, and

of the correct phase to modify the phase of the rotor currents, or, in other words, to produce in the rotor the current required to magnetise the stator. It is important to note that when the magnetising current is fed into the rotor from an auxiliary machine in this way, the voltage at which it is supplied is about equal to the main supply voltage multiplied by the slip of the motor. Thus the kVAr that have to be supplied to the rotor to magnetise the stator will be only about 5 per cent. of the kVAr taken from the main supply in the normal manner.

A phase advancer contains an armature with a commutator supplied with three sets of brushes which can be connected in circuit with the rotor of the main induction motor to which the armature is direct-coupled or driven by belting or gearing. The armature rotates in a core like the stator of an induction motor. The action of the phase advancer depends upon the fundamental principle that if an armature rotates in a magnetic field which, while being constant in magnitude, itself rotates in space, the frequency of the voltage at the stationary brushes will depend upon the rotational speed of the magnetic field relative to the brushes, and will be independent of the speed of the armature.

In the shunt phase advancer the core of the stator is provided with a 3-phase winding like that of an induction motor. Corresponding ends of the

three winding sections are connected to the brushes, and the other ends are joined in star through regulating resistances. This is known as the exciting winding, and as this winding is connected to the rotor of the main motor, the currents in it have the same frequency as those in this rotor, and the rotating magnetic field set up by the exciting currents moves therefore at a speed relative to the brushes which corresponds to the slip of the main machine. The frequency of the voltage generated at the brushes of the phase advancer therefore corresponds to that of the rotor currents of the main motor. The phase of the voltage generated by the phase advancer and injected into the rotor circuit of the main motor depends upon the position of the brushes relative to the stator windings, and, by suitable adjustment of the brush position the injected voltage can be made to produce in the rotor the current required to magnetise the main motor. The regulating resistance in the exciting windings on the stator controls the strength of the rotating field, and hence the magnitude of the injected voltage and of the magnetising kVAr. Thus the phase advancer with a given setting of the regulating resistance in the stator exciting winding produces magnetising kVAr, the magnitude of which is practically independent of the load on the main motor, and when the kVAr so produced is in excess of that required by the motor, this

excess is returned to the supply by transformer action in the main motor so that the power factor of the supply becomes leading. The shunt phase advancer is therefore somewhat like the direct current exciter of a synchronous motor.

It may, perhaps, clarify the action of a phase

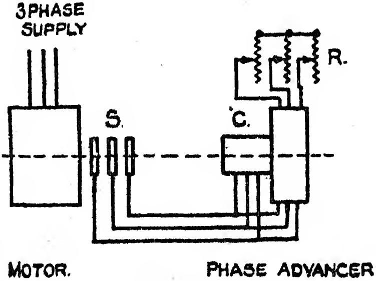

FIG. 27.

advancer if the following principles are recapitulated:

(*a*) The frequency of the voltage at the brushes depends only upon the speed, relative to the brushes, of the rotating field set up by the stator windings and hence upon the frequency of the exciting currents, and is independent of the speed of the armature and the strength of the field.

(*b*) The magnitude of the voltage of the brushes depends upon the speed of the armature relative to the field and upon the strength of the field.

Fig. 27 is a schematic diagram showing the connections of a phase advancer to a motor. The

motor is started in the usual way without the phase advancer and when normal speed is attained the slip rings S are connected to the commutator C of the phase advancer. R is the regulating resistance in the stator exciting windings of the advancer, for control of the lagging kVA generated by the auxiliary machine.

Power Factor Correction Motors.

A phase advancer and an induction motor can be combined in one machine as is done in the "No-Lag" motors manufactured by the British Thomson

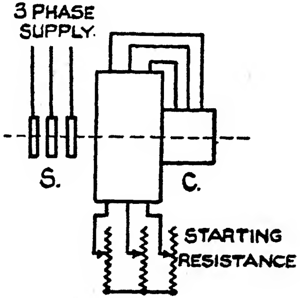

Fig. 28.

Houston Co. Ltd. (fig. 28). Machines of this type are virtually modified inverted induction motors, that is, the primary winding, usually in the stator, is in the rotor and is connected to the supply by brushes and slip rings S. The secondary winding is in the stator. The action of an inverted induction

motor is similar to one of the ordinary kind. The primary currents in the rotor set up a rotating field which moves at synchronous speed relatively to the rotor. The torque set up by the interaction of the secondary current and the field makes the speed of the field diminish and so causes motion of the rotor. When the machine has fully accelerated, the rotating field produced by the moving primary moves relatively to the stationary secondary at the slip speed, so that, as in the ordinary machine, the frequency of the secondary currents corresponds to the slip. The rotor of the "No-Lag" motor also carries an additional winding that is connected through a commutator, C, and brushes to the secondary winding on the stator. In accordance with the principle stated on page 122 the pressure at the brushes of this commutator winding will have a frequency corresponding to the speed of the field in which it rotates, and this frequency will consequently be the same as that of the secondary currents. The connection of the brushes to the stator winding therefore injects a voltage into the secondary circuit, and, as with a phase advancer, the phase of the injected voltage can be adjusted by varying the brush position so that the commutator winding generates the magnetising current required to produce the working field, and also, if required, additional magnetising current, which, transferred to the primary winding by transformer

action, can be returned to the supply for power factor correction. The magnetising kVAr so returned to the supply is practically independent of the power output of the motor, so that the leading power factor of the input to the machine falls as the load is diminished. The full load power factor of "No-Lag" motors may be about 0·9 leading when they give a magnetising kVAr output equal to about 50 per cent. of their kW consumption.

Motors of the "No-Lag" type can be used exclusively for the generation of magnetising kVAr, in which case they are called asynchronous condensers.

Static Condensers.

We have seen in Chapter I that a condenser, consisting of two metallic surfaces separated by a thin layer of insulation, contains a quantity of electricity, or charge, proportional to the pressure between these surfaces. If a condenser is connected to a source of alternating voltage, the continual change of the charge will result in an alternating charging current which is $\frac{1}{4}$ cycle in phase advance of the voltage. A condenser will therefore generate magnetising or lagging kVAr, and can be used for power-factor correction. A condenser of this kind when so used is generally called a static condenser in contradistinction to

rotary condensers which are rotating machines designed to produce lagging kVAr.

The alternating current passing into a condenser is proportional to its capacitance, and the combined capacitance of condensers in parallel is equal to the sum of the component capacitances.

Condensers for power-factor correction can therefore be designed to produce any required kVAr by connecting standard units in parallel. The condenser units are made up by winding three layers of paper interleaved with two layers of metallic foil, usually aluminium, on a former. The capacitance of a unit condenser may be about $4\frac{1}{2}$ microfarads. The required capacitance of the complete condenser is built up by assembling and fixing the requisite number of units on a board of insulating material and connecting the units in parallel by means of copper bus bars secured to the board. Each unit is protected by a small fuse. The complete assembly, which may consist of several sets of units, is fixed in a steel tank and is connected to the external terminals. The units are vacuum dried, and the tank is then filled with insulating oil, which reinforces the insulation and, by convection, dissipates the heat produced by the electrical loss in the whole condenser.

This loss is of the order of 0·3 per cent. of the kVAr rating of the whole condenser. Thus, a condenser designed to produce 100 kVAr of re-

active output will have a loss of about 300 watts. It is sometimes said that the efficiency of a static condenser is about 99·7 per cent., but a statement of this kind is not really accurate. A condenser gives no power output and it has no efficiency. The kW loss in a condenser is a different kind of physical quantity from the kVAr that it is designed to produce.

A complete static condenser built up in the way described can easily be fitted with more than two terminals to give several values of the effective capacitance. The extra terminals are connected in such a way that various numbers of units in parallel are included between the several pairs of terminals.

A large condenser will hold a considerable charge if it is disconnected from the supply at the instant the pressure is a maximum, and for some time after it is switched off it will have a considerable pressure on its terminals. For this reason a high discharge resistance is often connected permanently to the condenser terminals, to provide a leakage path for any charge that may be left in the condenser after switching off. This charge is dissipated in the leak, so that the pressure rapidly drops to a harmless value.

If reference is made to page 15, it will be understood that as the current in a condenser is proportional both to the pressure applied and also

to the capacitance, the reactive kVAr output of a condenser depends upon its capacitance and the square of the applied voltage. Thus the capacitance required per kVAr at 200 volts will be four times that required per kVAr at 400 volts. When power-factor correction is required at low pressures it is consequently sometimes economical to step-up the pressure applied to the condenser by means of an auto-transformer. The cost per unit of capacitance of a condenser is practically constant for pressures up to 600, so that by increasing the condenser pressure the cost of the condenser can often be largely reduced. This reduction in cost is, of course, obtained at the expense of the capital cost of the auto-transformer and of the additional losses in it. The question whether it is economical to raise the voltage on a power-factor correcting condenser is one which cannot be settled by fixed rules as it depends upon a number of circumstances peculiar to the actual power-factor problem. The maker of static condensers can, however, be relied upon to give advice on this matter.

Review of Methods of Power Factor Correction.

These methods fall into three distinct classes according to the class of machine or apparatus used: synchronous motors, induction motors with external or internal phase advancing, and static condensers.

Synchronous machines have the advantage that the reactive kVAr output can be adjusted to any desired value from zero to the rated output of the machine by variation of the direct current in the field windings. If a synchronous machine is used as a motor for power-producing purposes, it can, however, only correct power factor while it is actually required as a motor, unless, when not so required, it can be uncoupled from its load. A synchronous condenser used only for kVAr output, will always be available for the supply of any required reactive kVAr output, but the kW loss will be considerably greater per kVAr than with a static condenser, and this loss will not vary proportionally with the output, but will be relatively greater when this output is small.

Induction motors with internal or external phase advancers are generally suitable only for a practically fixed kVAr output. The reactive output of an asynchronous condenser can be varied, but not so conveniently as that of a synchronous condenser.

Static condensers have the great advantages of very small losses and almost negligible maintenance costs since they contain no moving parts. The reactive output of a static condenser is not, however, continuously variable, and only a few fixed values of the output can be obtained by subdivision of the condenser.

The Control of Power Factor.

Excepting in circumstances when the installation of a motor of a special type such as a synchronous or a variable speed commutator motor is otherwise desirable, power-factor correcting plant is used by power consumers with the object of reducing the charges for their supply. If the whole of the maximum reactive kVAr required by an installation of induction motors is produced by power-factor correcting plant but only a fraction of the total number of motors is running the kVAr output will be in excess of that required by the consumer and the balance will be returned to the supply mains. The overall power factor of the supply will become leading.

Supplies at leading power factors are generally disadvantageous to supply authorities, and power consumers who correct their power factor are desired by the supply authority to avoid returning reactive supply to the mains. From the point of view of the consumer, the production of unnecessary reactive kVAr involves unnecessary losses, and on general grounds it is desirable to avoid leading power factors of the supply.

We may say here that when the charge for a consumer's supply is based upon kVArh consumption, the meter registering lagging kVArh is generally fitted with a ratchet gear, so that, if the

reactive consumption reverses, and the power factor becomes leading, reverse registration of the meter is prevented. A consumer cannot therefore cancel a reactive consumption by returning lagging kVArh through the meter to the mains at times when he is not using the supply for power purposes.

Where a consumer is charged for his supply under a kVA maximum demand tariff, the control of his power factor may be a matter of considerable importance. Let us suppose that an uncorrected power factor of 0·5 at maximum load is raised to unity. The kVA per kW before correction will be 2, and the kVAr per kW will be $\sqrt{(2^2-1^2)}=\sqrt{3}=1{\cdot}73$, and this will be the kVAr output of the correcting plant. With the power factor corrected, the kVA demand will be equal to the maximum kW. If the power demand becomes zero, and the correcting plant continues its kVAr output, this, being 1·73 times the maximum corrected kVA value, will increase the indicated maximum demand. In a case like this, therefore, control of the power factor would be essential in order to prevent an increase of the indicated demand.

When static condensers are used for power-factor correction, the connection of part or all of the condenser capacitance to the supply mains can be controlled automatically by a relay responsive either to the power factor of the supply or to the power consumption, so that the condenser capaci-

tance in circuit is roughly suited to the power-factor correcting requirements. When one or more large motors are used, it is preferable to connect directly to the motor terminals separate condensers rated approximately to supply the magnetising kVAr of the motors. With this arrangement the condenser is automatically disconnected when the motor is shut down, and the production of excess kVAr is prevented. The supply of kVAr to the smaller motors may be obtained by an additional condenser connected to the main supply and controlled by an oil circuit breaker, either manually or automatically.

We have already referred to the ease with which the lagging kVAr output of a synchronous machine, and hence, the power factor of a whole supply can be controlled by the regulation of the direct current that provides the magnetism of the machine. Automatic regulation of this kVAr output is feasible, but being complicated is somewhat costly.

The details of the technique of the automatic regulation of power factor correcting devices are of a technical nature, and lie without the scope of this work. Such automatic regulation is only necessary in circumstances where there is a possibility of the kVA demand being increased by the accidental omission to reduce the kVAr output of correcting plant. Otherwise, manual control of

this output, either by regulation of the direct current supply to a synchronous machine, or by the hand switching of condenser sections will be sufficient.

Power-Factor Correction with Private Generating Plant.

When a consumer generates an A.C. supply to a group of induction motors by his own plant, the magnetising kVAr required by the motors may be produced by the generating plant, by power factor correcting apparatus, or by both. If the kVAr are produced by the generator, the current per kW of power output will be increased, and the windings of the machine must be suitable to carry this increased current corresponding to the full kW load. Further, the circulation of reactive current in the alternator windings produces a magnetic field that rotates synchronously with, and is in direct opposition to, the field that produces the pressure. To provide kVAr, therefore, the direct current supplied to the alternator must be increased, and this increases the size of the direct-current exciter required to give the requisite full load exciting current supply.

An alternator can be designed to give any kW and kVAr output simultaneously. The maximum kW output determines the size of the prime mover that drives the alternator: the simultaneous

maximum kW and kVAr outputs determine the rating of the alternator in kVA. A complete generating set, comprising prime mover and alternator is usually rated in terms of its maximum kW output and the minimum power factor at which this output can be sustained without overheating of the alternator. Thus a generating set rated at 1000 kW, 0·8 power factor, will comprise a prime mover capable of producing 1000 kW of electrical output and an alternator that will carry 1250 kVA continuously. Such a machine will produce at the full kW load a kVAr output that is 0·75 of this kW load, or 750 kVAr.

Although reactive kVAr with a private supply is produced most simply by the generating plant, it is not usual for A.C. generators to be rated for power factors lower than 0·8. If the power factor of the supply to the power installation does not fall below this value, the alternator will produce the kVAr required. If the load power factor is below that for which the generator is rated, this generator will be electrically fully loaded before the maximum output of the prime mover is reached. If, for instance, the load power factor is 0·5, the 1250 kVA full load of the alternator referred to in the preceding paragraph will correspond to only 625 kW and the remaining 375 kW of prime mover capacity will be wasted. The full load of 1000 kW at 0·5 power factor cannot be carried by the set

as a whole because the kVA output of 2000 would overload the alternator.

Whenever, therefore, the power factor of the load on a private A.C. plant is less than that for which the alternators are rated, the installation of power-factor correcting plant will increase the effective output of the generating plant by enabling both prime movers and alternators to be used to their full capacities. Power-factor correction will therefore be economical if the kVA load exceeds that of the alternators while the kW load is less than that of the generators, but not otherwise. As long as there is spare kVA capacity in the alternators, it will generally be most economical to allow them to generate the kVAr required.

The control of the power factor of a private supply when correcting plant is used may be even more important than it is when the supply is obtained from a public authority. In the first place, if the uncorrected power factor is very low, then the total kVAr capacity of the correcting plant required to raise this power factor to that for which the alternator is rated may be sufficient to overload the alternator, so that, the disconnection of part, at least, of the power-factor correcting kVAr capacity should be switched out as the kW load on the plant falls. A numerical example of this possibility will be found in the following chapter. When static condensers are used for

power-factor correction the risk of an excessive kVAr capacity remaining connected to the generating plant can be completely avoided by joining suitably proportioned sections of the total condenser capacitance directly to the terminals of the largest motors, so that these sections are automatically disconnected when the motors are shut down.

A second and even more important reason of the necessity of power factor regulation arises from the circumstance, already mentioned, that the return of lagging kVAr to an alternator reduces the direct current required to maintain the normal voltage. The regulation of the direct current in alternators of moderate rating is often carried out by means of a variable resistance in the magnetising circuit of the exciter supplying this current, whereby the voltage of the exciting machine is varied. This method of controlling the alternator pressure is quite satisfactory provided the amount of exciting current required does not fall much below that required for normal pressure at no load. If, because of the effect of the return of lagging kVAr to the alternator from the power-factor correcting plant, the direct current required falls to a value much below this, regulation of the exciter voltage will become difficult, and this voltage may become unstable, so that a small movement of the regulating resistance may cause

a very large variation of the alternator pressure. Otherwise the effect of the excessive kVAr returned to the alternator may have so great a magnetising effect that it becomes impossible to reduce the alternator pressure to the correct value by the regulating resistance, even if this resistance is connected in the direct current circuit of the alternator and the exciting voltage is practically constant. For this reason some means of adjustment of the kVAr produced by power-factor correcting plant is generally essential when A.C. is produced by a private plant, and in the design of an installation of static condensers full attention must be paid to this matter, and suitable switchgear must be provided for the control of the condenser kVAr connected to the supply.

CHAPTER VI

POWER FACTOR CALCULATIONS

Numerical Examples.

In this chapter we shall illustrate the principles of power-factor correction in a concrete way by some fully-worked numerical examples.

Our first example will be a typical simple power-factor correction problem which will be worked in three distinct ways:

Find the kVAr of condenser capacity required to raise the power factor of a load of 120 kW from 0·6 to 0·95.

This problem can be solved graphically in the following way. Draw a line OW (fig. 29) of 10 convenient length units to represent to scale 1 kW. On OW draw the semicircle OAW. In the semicircle place two lines, OA equal to 10 multiplied by the uncorrected power factor, *i.e.* $10 \times 0{\cdot}6 = 6$ in length units, and OB equal to 10 multiplied by the corrected power factor, *i.e.* $10 \times 0{\cdot}95 = 9{\cdot}5$ in length units. Draw WX_2 and produce OA and OB to meet the line in X_1 and X_2 respectively. Then the length X_1X_2 gives the correcting kVAr per kW required to the same scale as OW represents

1 kW. If the construction is carried out, the length X_1X_2 is found to be approximately 10 length units, and the kVAr per kW required is approximately 1. The total capacity of the correcting condenser must therefore be $1 \times 120 = 120$ kVAr.

The correctness of this construction is easily seen. As the two angles OAW and OBW in the

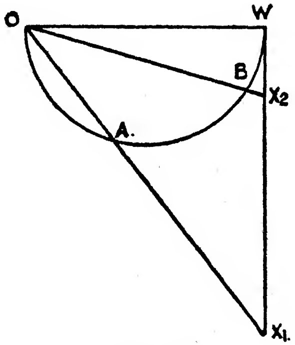

FIG. 29.

semicircle are right angles, the ratio OW/OA is equal to the ratio OX_1/OW and as OA/OW is 1 divided by uncorrected power factor, OX_1 is equal to kVA per kW, and WX_1 is equal to the kVAr per kW, in the original load to the same scale as OW is equal to 1 kW. Similarly the kVAr in the supply after the power factor has been raised is represented by the length WX_2. To reduce the kVAr in the supply from WX_1 to WX_2, the con-

densers must supply kVAr per kW represented by X_1X_2 to the same scale as OW represents 1 kW.

Alternatively the problem may be solved by the use of trigonometrical tables. Power factor normally represents the cosine of an angle of phase difference, and the tangent of this angle is numerically equal to the kVAr per kW at the power factor corresponding. The uncorrected power factor of 0·6 corresponds to a phase angle of about 53·1 degrees, and the tangent of this angle is 1·333. Similarly the corrected power factor of 0·95 corresponds to a phase angle of about 18·2 degrees, and the tangent of this angle is 0·327. To raise the power factor of the supply from 0·6 to 0·95, the kVAr to be supplied by the condenser must be $1{\cdot}333 - 0{\cdot}327 = 1{\cdot}006$ per kW. The total capacity for 120 kW is therefore $120 \times 1{\cdot}006 = 120{\cdot}7$ kVA.

The direct solution of the problem by first principles is carried out as follows:

At 0·6 power factor the kVA in 120 kW $= 120/0{\cdot}6 = 200$.

The kVAr $= \sqrt{(200^2 - 120^2)} = \sqrt{(25{,}600)} = 160$.

At 0·95 power factor the kVA $= 120/0{\cdot}95 = 126{\cdot}3$.

The kVAr $= \sqrt{(126{\cdot}3^2 - 120^2)} = \sqrt{(1550)} = 39{\cdot}4$.

The condenser capacity to reduce the kVAr consumption from 160 to 39·4 will be $160 - 39{\cdot}4 = 120{\cdot}6$.

As our second example we shall calculate the actual capacitances of the condensers required in the last example, say 120 kVAr, if the supply is

200 volts, 3 phase, 50 cycles, first, if the condensers are connected directly to the supply, and secondly, if the voltage on the condensers is raised to 600 volts by transformers.

Condensers for 3-phase power-factor correction are divided into three equal sections, each section being connected between two lines of the supply. The 120 kVAr of capacity will thus be divided into three sections each having a capacity of 40 kVAr.

The current in each 40 kVAr section at 200 volts line pressure will be $40,000/200 = 200$ amperes.

The current into a condenser $= 2\pi \times$ frequency $\times$ voltage $\times$ capacitance in farads.

$$\text{Thus, farad capacitance} = \frac{200}{2\pi \times 50 \times 200}$$

$$= \frac{1}{314}.$$

Or, expressing capacitance in the more usual microfarad units, each section will have a capacitance of

$$\frac{1,000,000}{314} = 3180\ \mu\text{F}.$$

With a condenser voltage of 600, the condenser current will be $40,000/600$ amperes, and microfarad capacitance will be

$$\left(1,000,000 \times \frac{40,000}{600}\right) \div (2\pi \times 50 \times 600)$$

$$= \frac{1,000,000 \times 40,000}{600 \times 600 \times 314} = \frac{1,000,000}{9 \times 314} = 353\ \mu\text{F}.$$

The required condensers will therefore consist of three sections each of 3180 μF capacitance at 200 volts, or each of 353 μF at 600 volts.

This result agrees with the rule given on page 128, that the capacitance required per kVAr of reactive output is inversely proportional to the square of the condenser voltage.

In our third example we shall find the condenser capacity which when directly connected to the terminals of a 50 h.p. induction motor will raise the full load power factor from 0·9 to unity. We assume that the full load efficiency is 92 per cent.

The output at full load $= 50 \times 0{\cdot}746$ kW.

The input at full load $= \dfrac{50 \times 0{\cdot}746}{0{\cdot}92} = 40{\cdot}5$ kW.

The kVA input at full load

$$= \frac{40{\cdot}5}{\text{power factor}} = \frac{40{\cdot}5}{0{\cdot}9} = 45.$$

The kVAr input at full load

$$= \sqrt{(45^2 - 40{\cdot}5^2)} = \sqrt{380} = 19{\cdot}5.$$

To raise the input power factor to unity, 19·5 magnetising kVAr must therefore be produced by the correcting condenser.

We shall next find the kVA rating of a 100 h.p. synchronous motor that, when running at full load in parallel with another supply of 150 kW at 0·6 power factor, will make the overall power factor

0·95 lagging. The efficiency of the synchronous machine will be taken as 92 per cent.

At 0·6 power factor the kVA in 150 kW is 150/0·6 = 250.

The kVAr $= \sqrt{(250^2 - 150^2)} = 200.$

The power output of the synchronous motor at full load is $100 \times 0{\cdot}746 = 74{\cdot}6$ kW.

The power input is $74{\cdot}6/0{\cdot}92 = 81{\cdot}5$ kW.

At the corrected power factor of 0·95 with the synchronous motor running, the total kW from the supply will be $150 + 81{\cdot}5 = 231{\cdot}5$.

The kVA in this total supply $= 231{\cdot}5/0{\cdot}95 = 244$.

And the lagging kVAr taken from the supply $= \sqrt{(244^2 - 231{\cdot}5^2)} = 75$.

The synchronous motor must therefore reduce the kVAr from the supply from 200 to 75, and it must therefore produce 125 kVAr.

The kVA in the motor supply will therefore be $\sqrt{(81{\cdot}5^2 + 125^2)} = 149{\cdot}5$, and the power factor of this supply will be $\dfrac{\text{kW}}{\text{kVA}} = \dfrac{81{\cdot}5}{149{\cdot}5} = 0{\cdot}545$ leading.

The graphical solution of this problem is given in fig. 30. Here the right-angled triangle OAB is drawn so that OA represents to scale 150 kW in the original load, and OB the kVA, equal to $150/0{\cdot}6 = 250$. The triangle OCD applies to the final load; OC represents to scale the final kW value of 150 plus the synchronous motor input of 81·5, and OD is equal to this 231·5 kW value

divided by the final power factor of 0·95. The length of ED represents the amount by which the kVAr from the supply are reduced by the synchronous motor, and as BE represents the kW input to this motor, the kVA supply to it is given to scale by BD.

We shall now work a problem illustrating the point mentioned in the final section of Chapter V.

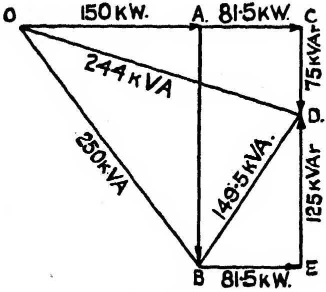

FIG. 30.

An alternator rated at 200 kW, 0·8 power factor, runs fully loaded to give a supply the power factor of which is raised from 0·4 lagging to 0·8 by correcting plant. If the kW load on the generator falls to $\frac{1}{10}$ the full load value, and the power factor of the supply remains 0·4, find the kVA load on the machine with the whole of the power factor correcting plant connected to the system.

At 0·4 power factor the kVA per kW in the supply is $1/0{\cdot}4 = 2{\cdot}5$.

The kVAr per kW is $\sqrt{(2{\cdot}5^2 - 1)} = 2{\cdot}289$.

At 0·8 power factor the kVA generator output is $1/0{\cdot}8 = 1{\cdot}25$ per kW.

The kVAr per kW in this output is $\sqrt{(1{\cdot}25^2 - 1)} = 0{\cdot}75$.

The kVA of correcting plant per kW to raise the power factor from 0·4 to 0·8 is therefore $2{\cdot}289 - 0{\cdot}75 = 1{\cdot}539$.

The total correcting capacity per 200 kW is therefore $200 \times 1{\cdot}539 = 307{\cdot}8$ kVA.

If the supply falls to $\frac{1}{10}$ of full load at 0·4 power factor,

The kW in this supply will be 20.

The kVAr will be $20 \times 2{\cdot}289 = 45{\cdot}8$.

The kW generator output will be 20.

The correcting plant will return to the generator $307{\cdot}8 - 45{\cdot}8 = 262$ lagging kVAr, and this will be the leading kVAr generator output.

The kVA generator output will therefore be $\sqrt{(20^2 + 262^2)} = 264$ and, as the full load kVA of the alternator is $200/0{\cdot}8 = 250$, it will be overloaded.

The graphical solution of this problem is shown in fig. 31.

The triangle OAC applies to the normal full load supply. OA is drawn to scale to represent 200 kW and OC to represent the 500 kVA with a power factor of 0·4. The triangle OAB represents to scale the full load generator output, OB being equal to $200/0{\cdot}8 = 250$ on the kW scale. BC

represents to scale the kVAr of correcting capacity to raise the full load power factor from 0·4 to 0·8. The triangle OED represents to scale one-tenth of the full-load supply at 0·4 power factor, of which

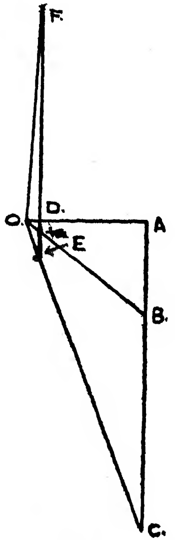

FIG. 31.

DE is the lagging kVAr component. EF equal in length to BC is the kVAr output of the correcting plant that remains working. The excess of EF over DE, the kVAr required to the supply is returned to the alternator, and represents its leading kVAr output. OF the remaining side of the triangle ODF is therefore the resultant of the

kW and kVAr outputs of the machine, and represents to scale the kVA load.

In our final numerical example we shall investigate the saving effected by power-factor correcting plant when the charges for the supply are regulated by a kVA demand tariff.

A consumer is supplied under a maximum demand tariff of £2, 10s. 0d. per kVA per year. Assuming the total capital maintenance and running costs of power-factor correcting plant are at the rate of £0·7 per year per kVA installed, find the net saving per kW of maximum demand by raising the power factor of the supply from 0·6 to (*a*) 0·9, (*b*) 0·95, (*c*) unity.

The kVA of demand

At 0·6 power factor	$=1/0{\cdot}6$	$=1{\cdot}667$	
,, 0·9 ,,	$=1/0{\cdot}9$	$=1{\cdot}111$	
,, 0·95 ,,	$=1/0{\cdot}95$	$=1{\cdot}053$	
,, unity ,,	$=$	$1{\cdot}000.$	

The corresponding annual demand charges per kW are, in shillings,

At 0·6 power factor	$1{\cdot}667 \times 50 = 83{\cdot}3$
,, 0·9 ,,	$1{\cdot}111 \times 50 = 55{\cdot}5$
,, 0·95 ,,	$1{\cdot}053 \times 50 = 52{\cdot}6$
,, unity ,,	$1 \times 50 = 50{\cdot}0.$

The kVAr per kW of maximum demand will be:

At 0·6 power factor	$\sqrt{(1{\cdot}667^2 - 1)} = 1{\cdot}333$
,, 0·9 ,,	$\sqrt{(1{\cdot}111^2 - 1)} = 0{\cdot}484$

At 0·95 power factor $\sqrt{(1{\cdot}051^2 - 1)} = 0{\cdot}329$
,, unity ,, $= 0.$

The kVAr of correcting plant per kW will be:

For 0·9 power factor $1{\cdot}333 - 0{\cdot}484 = 0{\cdot}849$
,, 0·95 ,, $1{\cdot}333 - 0{\cdot}329 = 1{\cdot}004$
,, unity ,, $1{\cdot}333 - 0 = 1{\cdot}333.$

The annual costs per kW of demand for these capacities of correcting plant, in shillings will be:

For 0·9 power factor $0{\cdot}849 \times 14 = 11{\cdot}9$
,, 0·95 ,, $1{\cdot}004 \times 14 = 14{\cdot}0$
,, unity ,, $1{\cdot}333 \times 14 = 18{\cdot}7.$

We can now collect our results and exhibit them in tabular form.

Power Factor of Supply.	Annual Charges per kW of Max. Demand in Shillings.			Annual Saving kW by Correction, in Shillings.
	kVA Demand.	Correcting Plant.	Total.	
0·6	83·3	..	83·3	..
0·9	55·5	11·9	67·4	15·9
0·95	52·6	14·0	66·6	16·7
Unity	50·0	18·7	68·7	14·6

These results show clearly that the net savings obtained by power-factor correction do not, under a maximum kVA demand tariff, increase continuously as the power factor is raised to its

maximum value of unity. The net saving in the whole power bill, including the expenses of correction, is greater at a final power factor of 0·95 than it is if additional correcting capacity is installed to bring the power factor up to unity. There is evidently some optimum value of the power factor that gives the maximum saving. We shall investigate this point more fully in the following section.

Power Factor Economics.

The object of what we have called power-factor tariffs is to make some kind of equitable charge for the reactive kVArh consumption by users of an A.C. supply, and thus to offer financial inducement for the production of this kVAr consumption by plant installed at the consumer's expense. The consumer will only install such plant if, after paying all charges incident to this installation, his total charges are reduced. Further, he will desire to maximise his net gains; in other words, to adjust his capital expenditure on power-factor correcting plant so that the overall charges for his supply are the least possible, and to avoid expenditure the charges incident to which are less than the consequent saving on the power bill.

The economical limit of power-factor correction, or the capacity of correcting plant that will maximise the overall savings, depends not only upon

the consumption and uncorrected power factor, but also upon the nature of the tariff under which the consumer is charged for the supply.

The simplest kind of power factor tariff is that under which the charge for the supply depends upon the kWh and the kVArh consumptions directly, kWh being charged for at one rate, and kVArh at another and a smaller rate. The problem of the economical limit of power factor correction in this case seems simple. If the consumer can produce his kVArh by his own plant at a cheaper rate than he can buy it from the supply authority, he will do so, and raise the average power factor of his supply to unity. 1 kVAr of correcting capacity continuously in service will produce 8760 kVArh a year. Taking the overall charges per kVArh of correcting capacity at the high value of 14s. a year, used in the numerical example on page 147 the cost of producing the reactive consumption will be $14 \times 12/8760$ or about 0·02d. per kVArh, a rate lower than would be quoted under any power-factor tariff.

The matter is not quite so simple as this however. It is not sufficient for the consumer to produce kVArh. He must obtain credit for this production in the registration of his kVArh meter. We have explained on page 130 that kVArh meters are usually provided with a ratchet gear to prevent backward registration of the meter when the supply

factor becomes leading, and lagging kVArh are returned to the supply. It follows therefore that, with such a meter, the consumer can only obtain credit for his power-factor correction when his plant is working, and further, that if he produces kVArh at a greater rate than that at which they

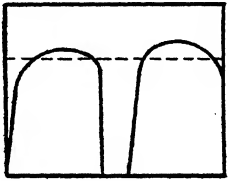

FIG. 32.

are used, the excess will, as far as his power bill is concerned, be wasted.

Suppose that over a day's use of the supply the kVAr load varies according to a curve like fig. 32. Now the area under the curve is proportional to the kVArh consumption over the day, and correcting plant having a capacity represented by the average height of the curve will during the day produce as many kVArh as are consumed. The dotted line in fig. 32 represents the constant rate of production of kVArh and it is seen at once that at the times this dotted line is above the curve, the correcting plant is returning kVArh to the supply, and as the kVArh meter is prevented from rotating backwards, this return is wasted as far as the consumer is

concerned. During the periods when the curve lies above the dotted line there is a net kVAr load on the system, and the kVArh metered consumption is increasing. It follows, therefore, that if a consumer wishes to annul or cancel the whole of his kVArh consumption, the capacity of the correcting plant required must be equal to the maximum kVAr in his supply, and as this maximum reactive load will rarely be known, complete correction of the power factor by metered consumptions will rarely be possible.

The average kVAr load can easily be determined by dividing a year's or a quarter's reactive consumption by the hours during which the plant is working at normal load. Thus, if the normal working week is 48 hours, then the hours of working in a 50-week year will be 2400; and a year's kVArh consumption divided by 2400 will give a value of kVAr of correcting plant that will prevent the greater part, but not the whole of the kVAr meter registration, assuming that the meter is prevented from registering backward. Probably if the averaging time be taken at the lower value of 2000 hours, the average kVAr so obtained will be sufficient to cancel practically all the reactive consumption. 1 kVAr of correcting plant will cancel 2000 kVAr of reactive consumption a year, and taking the charges a year at the high figure of 14s. the cost per kVAr so produced will be $12 \times 14/2000 = 0{\cdot}085$d.,

a rate which will usually be less than the tariff rate of supply.

The economical aspect of power-factor correction under a composite kW and kVArh tariff therefore depends upon the estimated cost of production of kVArh by correcting plant, and this estimate depends upon the estimated ratio of maximum to average kVAr loading. It will generally be impossible to estimate the correcting capacity required completely to prevent registration of a non-reversible reactive meter, but it will usually be feasible to prevent the greater part of this consumption with a substantial net gain. The actual economical limit of power-factor correction depends upon the unknown shape of the kVAr load curve of the consumer, and this limit is consequently indeterminate.

When a supply is charged for under a two-part maximum kVA demand tariff, the economical limit of power-factor calculation by static condensers is readily calculable if the annual charges incident to the provision of correcting plant can be assumed to be proportional to its kVAr capacity. Fig. 33 shows a graphical construction for obtaining this economical limit.

Draw OA to scale to represent the annual demand charge per kVA. Complete the right-angled triangle OAB such that OA/OB is equal to the uncorrected power factor. OB then repre-

sents to scale the demand charge per kW at this power factor. Draw the semicircle OCA, and in it place the line CA representing to scale the annual charges per kVAr of power-factor correction. Draw OC, and BD at right angles to OC produced and meeting it at D. Now let the power factor

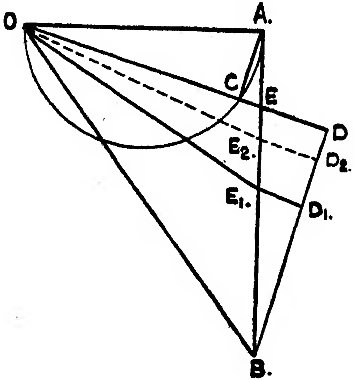

FIG. 33.

be increased by correcting plant so that it takes the higher value represented by the ratio OE_1/OA. Now if OA also represents 1 kW, AB represents the kVAr per kW in the actual load, AE_1 the kVAr in the supply at the new power factor and BE_1 the kVAr of correcting plant to increase the power factor. Now the ratio of the charge per kVAr of correcting capacity to the charge per kVA of demand is equal to AC/OA and this is equal to BE_1/E_1D where E_1D is drawn at right angles to

BD. The total charges per kW at the new power factor are therefore equal to the sum of the lengths of OE_1 and E_1D, OE_1 representing the demand charge and E_1D_1 the correcting charge, and it is easy to see that these total charges have the smallest value when the power factor has the value OA/OC, and the two lines OE and ED representing the component charges have the same direction. It is also seen from this construction that the power factor giving the smallest total annual charges per kW depends only on the ratio of the correcting and the demand charges, and that it is independent of the uncorrected power factor. It is further seen that the total charges vary very little as the corrected power factor changes from the theoretically economical value by small amounts on either side. The combined length of OE_2 and E_2D_2 for instance is very nearly the same as that of OD which represents to the money scale the minimum total annual charges per kW of maximum demand, excluding the kWh charges.

It will be understood that, as the charges in respect of the kWh consumption are unaffected by power-factor consumption, these charges are irrelevant to the preceding discussion.

We can by this geometrical construction calculate the most economical power factor that corresponds to the data used for the numerical example on page 147. In fig. 33, OA representing the demand

charge of 50s. will have a length of 50 units, and AC, representing the correcting charge, will have a length of 14 units. The most economical power factor OA/OC = OC/OA and as $OC = \sqrt{(50^2 - 14^2)} = 48$, this power factor is $48/50 = 0{\cdot}96$. The total charges at this power factor are almost exactly the same as at 0·95, already worked out.

A concise and more complete working of this power factor problem by trigonometry will be found in the Appendix.

The circumstance pointed out above, that small variations of the corrected power factor on each side of the calculated economical value make very little difference in the total annual charges, is of considerable practical importance. From the uncorrected and the economical power factor values and the maximum demand, the required correcting capacity can be worked out by the methods already explained and the cost of the correcting plant estimated. If this cost is a little greater than is convenient to the consumer, practically all the possible gain will be obtained by a moderate reduction of the correcting capacity. If, on the other hand, the consumer contemplates future extensions to his power plant, then the installation of extra kVAr of correcting capacity to meet the new conditions will entail very small increased charges in the meantime, provided, of course, that this additional capacity is not in excess of the

actual kVAr in the maximum kVA demand, and the power factor of this demand is not made leading.

Further, the cost of correcting plant will not generally be exactly proportional to its kVAr rating. A small increase in the capacity of a static condenser installation may, for instance, require switchgear and tank capacity of the next standard size, and so increase the cost of installation more than proportionally. In such a case it would probably be economical to choose the smaller rating, as the increased kVAr required for the nominally economical power factor might entail additional charges not offset by the very small reduction in the demand charge.

It can easily be seen from fig. 33 that the economical power factor rises as the ratio of correcting charges to demand charges falls. The ratio of 14/50 used in the numerical example is, at the time of writing, well above the average, when power-factor correction is effected by static condensers. There is no concise way to estimate an economical limit when power-factor correction is by motors delivering mechanical load.

The economical limit of power factor correction is also difficult to work out in an elementary way when the supply charges are regulated by a sliding-scale depending upon average power factor in the manner explained on page 100. A short mathe-

matical investigation of this problem will be found in the Appendix, from which it will appear that, when power-factor correction is carried out by static condensers, it will usually pay the consumer to produce the greater part of his kVAr consumption and to raise his measured average power factor to a value of the order of 0·95.

APPENDIX

Distorted Current Waves. (See page 24.)

An alternating current of distorted or non-sinusoidal wave form is, by Fourier's Theorem, the resultant of a number of harmonic components, the frequencies of which are integral multiples of the nominal frequency. If the amplitudes of the second, third, and higher harmonic components bear ratios of n_2, n_3, n_4 . . . to the amplitude of the first harmonic or fundamental component, and if the R.M.S. value of the fundamental is I_1 the R.M.S. value of the resultant current is

$$I_1\sqrt{(1+n_2^2+n_3^2 \ldots)}$$

since the products of harmonic terms of different frequencies have average values of zero.

If the distorted current is produced by a sinusoidal voltage of R.M.S. value V, applied to a non-linear impedance, the power will be $VI_1 \cos \phi_1$ where ϕ is the angle of phase difference between the fundamental component of the current and the voltage; the harmonic components give no average power in association with the sinusoidal voltage of different frequency.

The volt-amperes are $VI_1\sqrt{(1+n_2^2+n_3^2\ .\ .\ .)}$ and the power factor

$$\frac{VI_1 \cos\phi_1}{VI_1\sqrt{(1+n_2^2+n_3^2\ .\ .\ .)}}=\frac{\cos\phi_1}{\sqrt{(1+n_2^2+n_3^2\ .\ .\ .)}}.$$

The quantity $1/\sqrt{(1+n_2^2+n_3^2\ .\ .\ .)}$ is equal to μ the distortion factor. $\cos\phi_1$ is the displacement factor. μ is the maximum value of the power factor.

The quantity $\sqrt{(1+n_2^2+n_3^2\ .\ .\ .)}$ is sometimes called the curve factor and $\sqrt{(n_2^2+n_3^2\ .\ .\ .)}$ the harmonic factor.

Average Power Factor. (See page 33.)

If the kWh and the kVArh consumptions in a supply are separately measured and simultaneous readings of the two meters are taken for small consumption increments, a curve like fig. 34 can be plotted, in which every point defines two simultaneous meter readings. Consider a small advance, $p\ q$, of the kWh meter. Erecting ordinates this defines the small triangle *abc* bounded by the curve, whereof *ab* and *bc* are simultaneous kWh and kVArh advances. Assuming the power factor is constant during this advance, the corresponding increment of kVAh consumption will be equal to the third side *ac* of the triangle, or to the length of the part of the curve intercepted between the ordinates on p and q. Thus the whole length

of the curve represents kVAh to the same scale as horizontal and vertical ordinates represent kW and kVArh.

The average power factor based on kVArh and kWh consumptions over the period corresponding to the curve will evidently be equal to the ratio AB/AC. If the kVAh consumption were measured

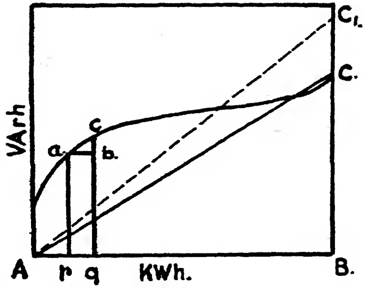

FIG. 34.

directly this would be represented by the length of the curve which is generally greater than AC. Let AC_1 be a straight line equal in length to the curve. Then the power factor based on the ratio kWh/kVAh is equal to AB/AC_1, and this latter ratio must be less than the power factor based on kWh and kVArh consumptions, excepting when the power factor over the whole period of consumption is constant, in which case the two values are equal.

3-Phase Volt-Amperes. (See page 32.)

The quantity "total equivalent volt-amperes" in a 3-phase circuit is illustrated in fig. 35. Here OA represents the watts and AD the VAr in one phase. OD will then represent the VA in this phase. The triangle DEF likewise represents the watts, VAr and VA in the second phase, and the

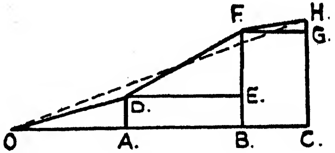

FIG. 35.

triangle FGH, these quantities in the third phase, the watt lines OA, DE and FG all being parallel. Completing the construction shown, it is seen that the total watts are represented by OC and the total VAr by CH, so that the total equivalent volt-amperes defined as $\sqrt{\{W^2 + (VAr)^2\}}$ are represented by the straight line OH. The arithmetical sum of the component values of the VA is evidently equal to the sum of the lengths of the lines OD, DF and FH and this sum is greater than OH representing the total equivalent volt-amperes, excepting when the component power factors are all equal. In a 3-wire supply this condition can only obtain if the load is balanced.

A third possible definition of 3-phase volt-amperes, referred to on page 57 is equal to $\sqrt{3} \times \sqrt{(A^2+B^2+C^2)}$ where A, B and C are the single-phase VA values represented in fig. 35 by the lines OD, DF and FH. The square of this third value is $3A^2+3B^2+3C^2$ and the square of the arithmetical sum of component VA values is

$$
\begin{aligned}
&(A+B+C)^2 \\
&\quad = A^2+B^2+C^2+2AB+2BC+2AC \\
&\quad = 3A^2+3B^2+3C^2-(A-B)^2-(B-C)^2-(A-C)^2
\end{aligned}
$$

which, as squares are essentially positive, must always be less than the square of the

$$\sqrt{3} \times \sqrt{(A^2+B^2+C^2)}$$

value excepting when $A=B=C$. The three values of 3-phase VA; total equivalent volt-amperes; arithmetical sum of component volt-amperes; and $\sqrt{3}$ times the square root of the sum of the squares of component volt-amperes, are therefore in ascending order of magnitude excepting when the 3-phase load is balanced, when they are all equal.

Current-operated 3-Phase Maximum Demand Indicator.

This instrument described on page 57 has three driving elements. In each element the driving torque is proportional to the current it carries. The motion of the rotor is resisted by two braking

torques, each proportional to the speed and one proportional to the square of the flux B in a permanent magnet and the other proportional to the square of the alternating flux that produces the driving torque. Assuming that this flux is proportional to the current, and neglecting friction, we have, equating driving and braking torques

$$a(I_1^2 + I_2^2 + I_3^2) = \text{speed} \times \{B^2 + b(I_1^2 + I_2^2 + I_3^2)\}$$

where I_1, I_2, and I_3 are the currents in the three elements.

Thus speed $= \dfrac{a(I_1^2 + I_2^2 + I_3^2)}{B^2 + b(I_1^2 + I_2^2 + I_3^2)}$.

The scale of the instrument is marked with equal currents in the three windings, and the advance of the pointer for the standard averaging time, being proportional to the speed, will also be proportional to

$$\frac{3aI_0^2}{B^2 + 3bI_0^2}$$

where I_0 is the value of the calibrating current. Thus the reading of the indicator with an unbalanced load will correspond to a calibrating current having the value $\sqrt{3} \times \sqrt{(I_1^2 + I_2^2 + I_3^2)}$, and the instrument, scaled to indicate kVA, will with unbalanced loads give a reading corresponding to $\sqrt{3}$ times the square root of the sum of the squares of the component kVA values.

Modifications of the Arno System of Charging.

We have seen on page 97 that according to this system the charge for the supply should be

$$C \times \text{kWh} + B \times \text{kVAh}$$

or if $B = \frac{1}{3}C$,

$$C\{(\text{kWh}) + \tfrac{1}{3} \times (\text{kVAh})\}.$$

If the power factor of the supply has an average value of $\cos\theta$, then for any other value $\cos\phi$, the quantity $\cos(\theta - \phi)$ will not differ from unity by more than 5 per cent. provided $\theta - \phi$ does not exceed ± 18 degrees. Thus the quantity upon which the charge is based is approximately

$$\begin{aligned}(\text{kWh}) &+ \tfrac{1}{3}(\text{kVAh}) \times \cos(\theta - \phi)\\ &= (\text{kWh}) + \tfrac{1}{3}(\text{kVAh})(\cos\theta\cos\phi + \sin\theta\sin\phi)\\ &= (\text{kWh}) \times \left(1 + \frac{\cos\theta}{3}\right) + (\text{kVArh}) \times \frac{\sin\theta}{3}.\end{aligned}$$

Thus, by assigning an average power factor $\cos\theta$, and finding $\sin\theta$, a quantity based on kWh and kVArh can be found which is very nearly the same as the quantity entering into the Arno system. If, for instance, the average power factor is taken as 0·707, $\sin\theta = 0{\cdot}707$ and the equivalent quantity is

$$(\text{kWh}) \times 1{\cdot}236 + (\text{kVArh}) \times 0{\cdot}236,$$

and if kWh and kVArh are charged for at respective rates of 1·236 C and 0·236 C, the total charge for a

supply will correspond to that under the Arno system.

Returning to the equivalent quantity

$$(\mathrm{kWh}) \times \left(1+\frac{\cos\theta}{3}\right)+(\mathrm{kVArh}) \times \frac{\sin\theta}{3}$$

this is equivalent to

$$(\mathrm{kVAh}) \times \left\{\cos\phi \times \left(\frac{1+\cos\theta}{3}\right)+\sin\phi \times \frac{\sin\theta}{3}\right\}$$

which by a well-known trigonometrical transformation becomes

$$\mathrm{kVA} \times \sqrt{\left\{\left(1+\frac{\cos\theta}{3}\right)^2+\left(\frac{\sin\theta}{3}\right)^2\right\}} \times \cos(\psi-\phi)$$

where
$$\psi = \text{arc tan}\ \frac{\sin\theta}{3}\bigg/\left(1+\frac{\cos\theta}{3}\right)$$
$$= \text{arc tan}\ \frac{\sin\theta}{3+\cos\theta}.$$

Thus the equivalent kW and kVAr consumption is equal to

$$\mathrm{kVAr} \times \sqrt{\left(1+\frac{2\cos\theta}{3}+\frac{1}{9}\right)} \times \cos(\phi-\psi).$$

If $\cos\theta = 0{\cdot}707$ then this expression is equivalent to

$$\mathrm{kVAr} \times 1{\cdot}26 \times \cos(\phi-\psi)$$

where $\psi = \text{arc tan}\ 0{\cdot}23 = 13$ degrees.

Now a 3-phase kWh meter can be compensated internally so that it integrates the quantity kVA $\times \cos(\phi - 13°)$ and $1{\cdot}26$ times the registration of the

meter would give a quantity closely approximating to the consumption used to determine the charge for the supply under the Arno system. It can be shown that this approximation is within $2\frac{1}{2}$ per cent. for a range of variation of ϕ, the angle determining power factor, of the order of 45 degrees.

Economical Limits of Power-Factor Correction. (See page 156.)

If A and B are respectively the annual charges per kVA of maximum demand and per kVAr of condenser correcting capacity, the total annual charges per kW for a power factor corrected from $\cos\phi_1$ to $\cos\phi$ under a two-part maximum kVA demand tariff will be:

$$\frac{A}{\cos\phi}+B(\tan\phi_1-\tan\phi).$$

Differentiating this expression with respect to ϕ and equating to zero we have, as the condition for minimum total charges,

$$\frac{A\sin\phi}{\cos^2\phi}-\frac{B}{\cos^2\phi}=0$$

whence $$\sin\phi=\frac{B}{A}$$

and this defines $\cos\phi$ the most economical power factor.

The minimum total charges are, putting $B=A\sin\phi$

$$A\left\{\frac{1}{\cos\phi}-\frac{\sin^2\phi}{\cos\phi}+\frac{\sin\phi\sin\phi_1}{\cos\phi_1}\right\}$$
$$=\frac{A}{\cos\phi_1}(\cos\phi\cos\phi_1+\sin\phi\sin\phi_1)$$
$$=\frac{A}{\cos\phi_1}\cos(\phi_1-\phi).$$

Thus, the minimum value of the annual costs per kW is equal to the demand charge $A/\cos\phi_1$ at the uncorrected power factor multiplied by the cosine of the angle $(\phi_1-\phi)$ of phase shift brought about by power-factor correction.

When the tariff rate is varied proportionally to the power factor change, according to a tariff like that described on page 100, it may be assumed the power-factor correction up to the value corresponding to the standard rate will be economical. For a power-factor $\cos\phi$ above this value $\cos\phi_1$, the rate per kWh will be reduced by the amount P $(\cos\phi-\cos\phi_1)$. Against the saving so obtained must be offset the cost of the kVArh required to raise the power factor which will be

$$Q(\tan\phi_1-\tan\phi) \text{ per kW},$$

where Q is the estimated cost of producing a kVArh by condenser plant. The net saving per kWh will therefore be

$$P(\cos\phi-\cos\phi_1)-Q(\tan\phi_1-\tan\phi).$$

Differentiating with respect to ϕ_1 and equating to

zero we have as the condition for the maximum saving:

$$-P \sin\phi + \frac{Q}{\cos^2\phi} = 0$$

whence $$\sin\phi \cos^2\phi = \frac{Q}{P}$$

and, knowing P and Q, the value of ϕ corresponding to the most economical power factor can be obtained by plotting values of the function $\sin\phi \cos\phi$.

In the tariff described on page 100 the kWh rate varies $\frac{1}{2}$ per cent. for each 1 per cent. increase of the power factor above 0·8. The constant P in the above working is therefore 0·5. On page 152 we showed that a value for Q, the rate of producing a kVArh by condensers would not be likely to exceed 0·085. The ratio P/Q will therefore usually be less than 0·5/0·085, say, not more than $\frac{1}{6}$. It is easy to see that, if this be the case, the condition for maximum economy is given very approximately by the formula

$$\sin\phi = \frac{Q}{P}$$

since an angle whose sine is $\frac{1}{6}$ has a cosine of 0·985, and the square of this cosine will differ from unity by only about 1 per cent.

TRIGONOMETRICAL FUNCTIONS

If cos ϕ is the power factor of an A.C. supply with sinusoidal wave forms, sec ϕ is the kVA per kW, and tan ϕ the kVAr per kW.

Deg.	Sin.	Cosec.	Tan.	Cotan.	Sec.	Cos.	
0	0	Infinite	0	Infinite	1·000 00	1·000 00	90
1	·017 45	57·298 7	·017 46	57·290 0	1·000 15	·999 85	89
2	·034 90	28·653 7	·034 92	28·636 3	1·000 61	·999 39	88
3	·052 34	19·107 3	·052 41	19·081 1	1·001 37	·998 63	87
4	·069 76	14·335 6	·069 93	14·300 7	1·002 44	·997 56	86
5	·087 16	11·473 7	·087 49	11·430 1	1·003 82	·996 19	85
6	·104 53	9·566 8	·105 10	9·514 4	1·005 51	·994 52	84
7	·121 87	8·205 5	·122 78	8·144 3	1·007 51	·992 55	83
8	·139 17	7·185 3	·140 54	7·115 4	1·009 83	·990 27	82
9	·156 43	6·392 5	·158 38	6·313 8	1·012 47	·987 69	81
10	·173 65	5·758 8	·176 33	5·671 3	1·015 43	·984 81	80
11	·190 81	5·240 8	·194 38	5·144 6	1·018 72	·981 63	79
12	·207 91	4·809 7	·212 56	4·704 6	1·022 34	·978 15	78
13	·224 95	4·445 4	·230 87	4·331 5	1·026 30	·974 37	77
14	·241 92	4·133 6	·249 33	4·010 8	1·030 61	·970 30	76
15	·258 82	3·863 7	·267 95	3·732 1	1·035 28	·965 93	75
16	·275 64	3·628 0	·286 75	3·487 4	1·040 30	·961 26	74
17	·292 37	3·420 3	·305 73	3·270 9	1·045 69	·956 30	73
18	·309 02	3·236 1	·324 92	3·077 7	1·051 46	·951 06	72
19	·325 57	3·071 6	·344 33	2·904 2	1·057 62	·945 52	71
20	·342 02	2·923 8	·363 97	2·747 5	1·064 18	·939 69	70
21	·358 37	2·790 4	·383 86	2·605 1	1·071 15	·933 58	69
22	·374 61	2·669 5	·404 03	2·475 1	1·078 53	·927 18	68
23	·390 73	2·559 3	·424 47	2·355 9	1·086 36	·920 50	67
24	·406 74	2·458 6	·445 23	2·246 0	1·094 64	·913 55	66
25	·422 62	2·366 2	·466 31	2·144 5	1·103 38	·906 31	65
26	·438 37	2·281 2	·487 73	2·050 3	1·112 60	·898 79	64
27	·453 99	2·202 7	·509 53	1·962 6	1·122 33	·891 01	63
28	·469 47	2·130 1	·531 71	1·880 7	1·132 57	·882 95	62
29	·484 81	2·062 7	·554 31	1·804 0	1·143 35	·874 62	61
30	·500 00	2·000 0	·577 35	1·732 1	1·154 70	·866 03	60
31	·515 04	1·941 6	·600 86	1·664 3	1·166 63	·857 17	59
32	·529 92	1·887 1	·624 87	1·600 3	1·179 18	·848 05	58
33	·544 64	1·836 1	·649 41	1·539 9	1·192 36	·838 67	57
34	·559 19	1·788 3	·674 51	1·482 6	1·206 22	·829 04	56
35	·573 58	1·743 4	·700 21	1·428 1	1·220 77	·819 15	55
36	·587 79	1·701 3	·726 54	1·376 4	1·236 07	·809 02	54
37	·601 82	1·661 6	·753 55	1·327 0	1·252 14	·798 64	53
38	·615 66	1·624 3	·781 29	1·279 9	1·269 02	·788 01	52
39	·629 32	1·589 0	·809 78	1·234 9	1·286 76	·777 15	51
40	·642 79	1·555 7	·829 10	1·191 8	1·305 41	·766 04	50
41	·656 06	1·524 3	·869 29	1·150 4	1·325 01	·754 71	49
42	·669 13	1·494 5	·900 40	1·110 6	1·345 03	·743 14	48
43	·682 00	1·466 3	·932 52	1·072 4	1·367 33	·731 35	47
44	·694 66	1·439 6	·965 69	1·035 5	1·390 16	·719 34	46
45	·707 11	1·414 2	1·000 00	1·000 0	1·414 21	·707 11	45
	Cos.	Sec.	Cotan.	Tan.	Cosec.	Sin.	Deg.

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